EPFL Cours d'Electrotechnique I

2. Conventions et les symboles :

→ concepts → modèle

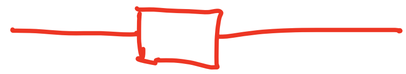
Ex :

courant : i, I, $\hat{I}$, $\hat{i}$, $\overline{I}$, $\underline{I}$

unité : [A]

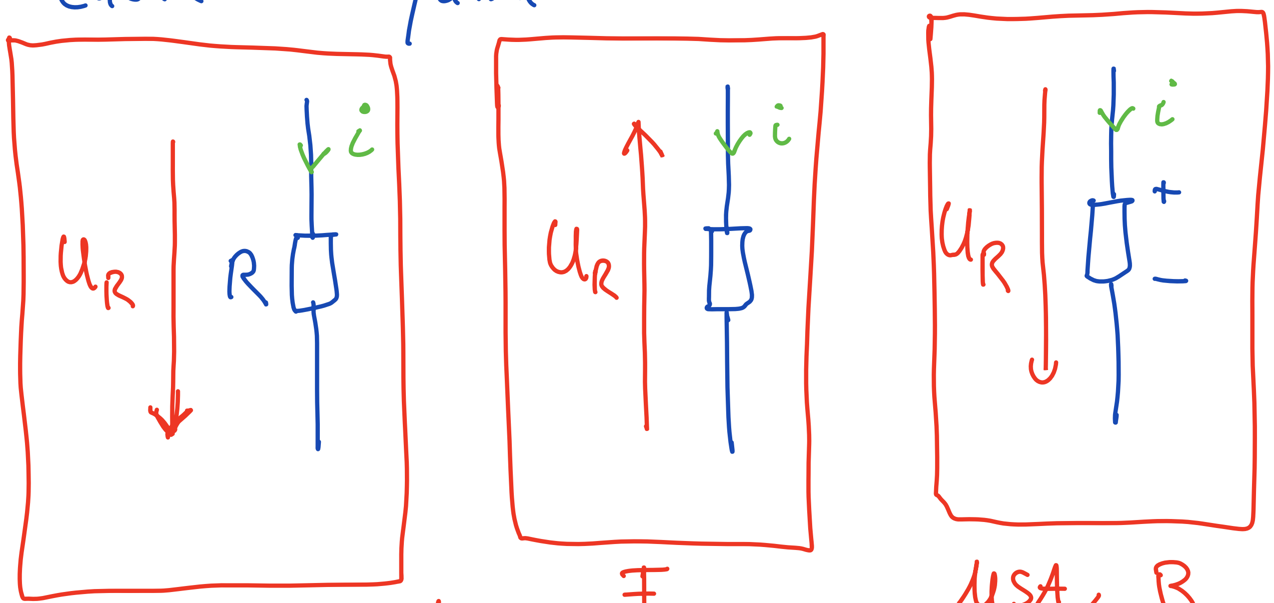
Relations : $U = R \cdot I$

$u = R \cdot i$

Dessin : 

Résistance

Choix à faire :

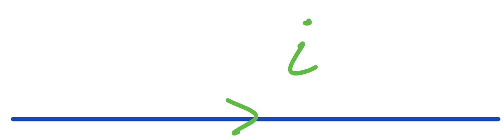


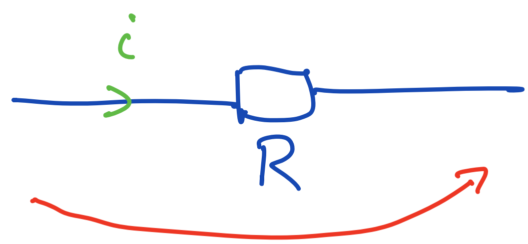
International   Fr   USA, B

convention moteur : choix

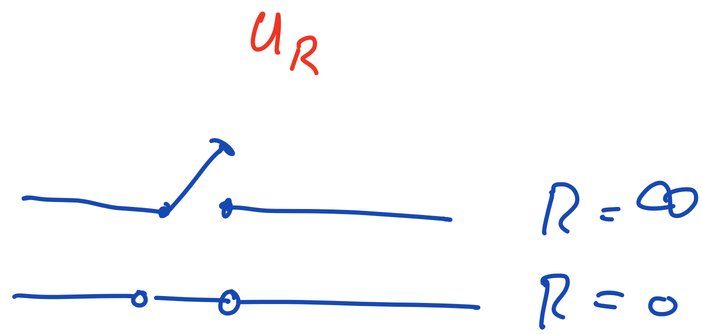
2.2 Représentation graphique :

Conducteur parfait :

conducteur avec un courant : 

Elément : 

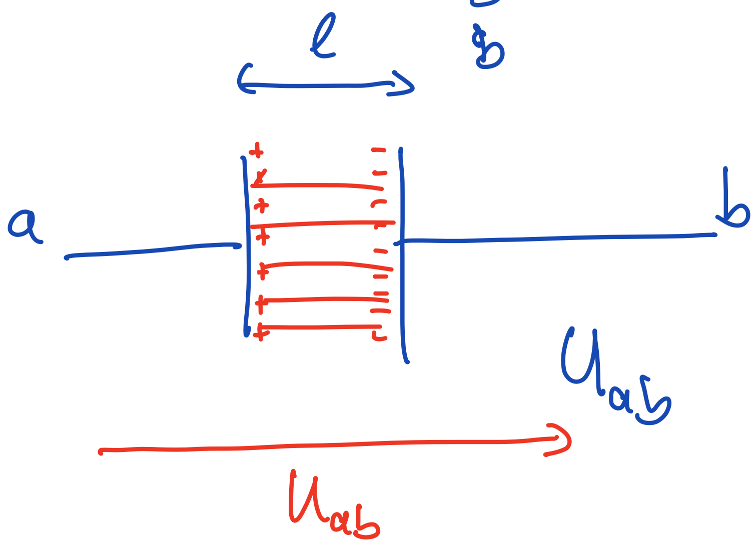
Interrupteur :



3. Lois fondamentales :

Différence de potentiel : Tension [Volt]

$$V_a - V_b = \int_b^a E\,dl = U_{ab} \quad [V]$$



3.2.19 La Capacité :

Définition : Charge électrique : Q

Capacité : $C = \dfrac{Q}{U_{ab}}$

Symbole : 

3.3 Courant électrique :

$$I = \frac{dQ}{dt} \quad [A]$$

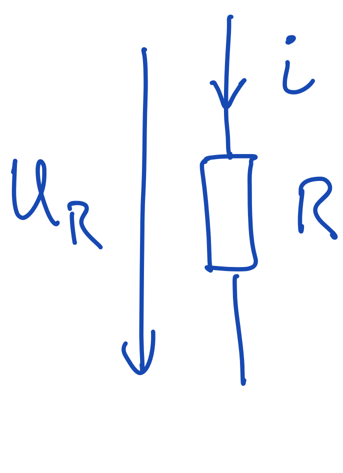
Densité de courant : $J \Rightarrow [A/m^2]$

3.3.4 Pertes Joule :

$$P = R \cdot I^2 \quad [W]$$

Récap :

Convention moteur :



→ Puissance positive

→ conv. moteur consommateur.

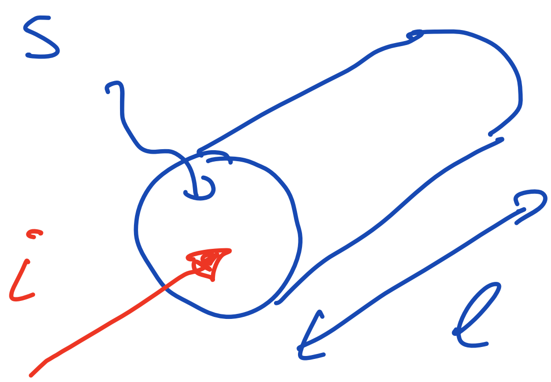
3.3.6 Définition de la résistance :

$$R_{ab} = \int_a^b \rho \cdot \frac{dl}{S}$$

dl : longueur

S : surface

ρ : résistivité électrique $[\Omega m]$



Si S est constante sur la longueur

$$R_{ab} = \frac{\rho \cdot l}{S} \quad [\Omega]$$

3.3.8 Loi d'Ohm :

$$U_{ab} = R_{ab} \, I$$

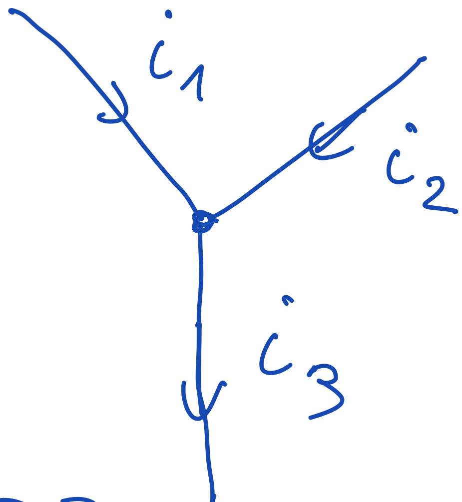
(courant et tension continue)

$u_{ab} = R_{ab} \cdot i$ (courant et tension variable)

3.3.11 Lois de Kirchhoff :

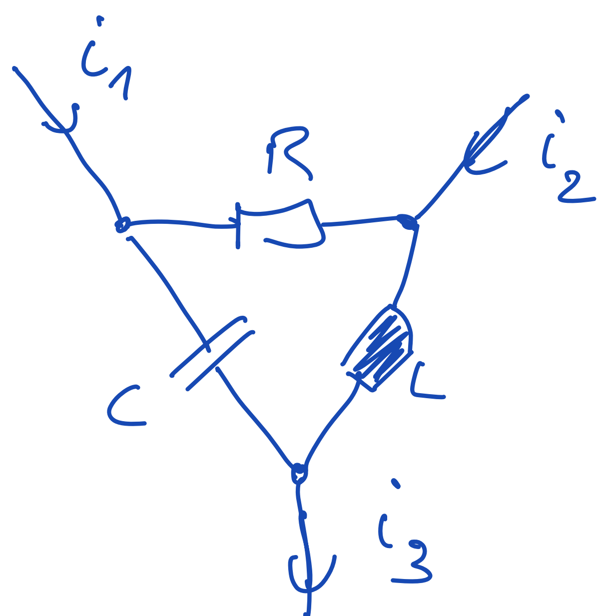
Noeud : Point de convergence d'au moins trois conducteurs

$$\underline{\sum i_j = 0}$$



$$i_1 + i_2 - i_3 = 0$$

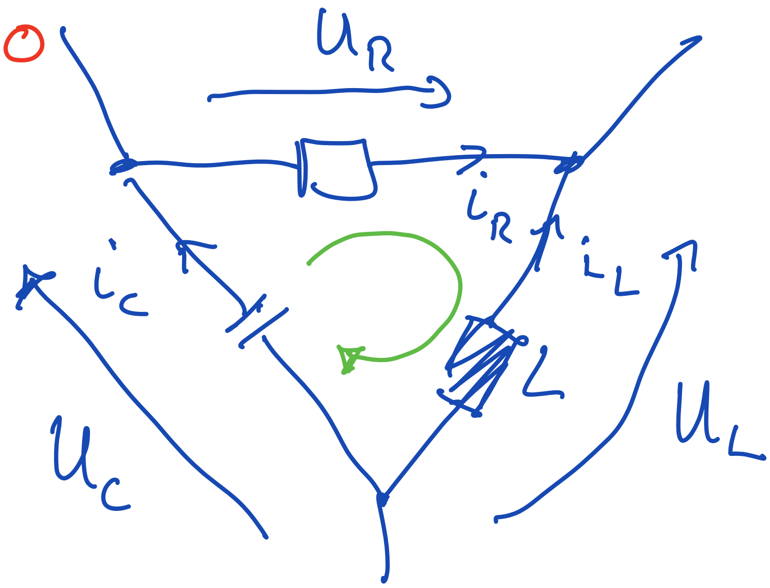
Noeud généralisé :



$$i_1 + i_2 - i_3 = 0$$

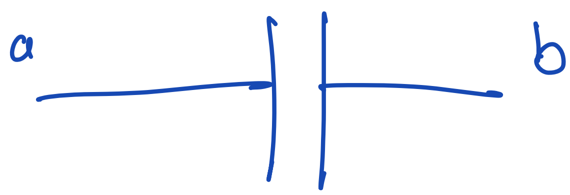
Maille : ensemble de branche partant d'un noeud pour y retourner

$$\sum U_j = 0$$



$$U_R - U_L + U_C = 0$$

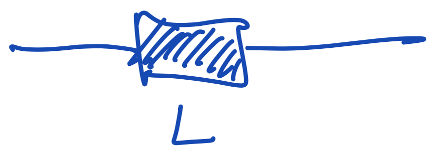
3.5 La Capacité :

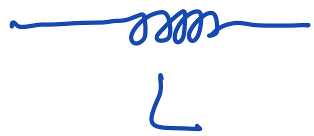


$$C = \frac{Q}{U_{ab}} \qquad Q = \int i \, dt$$

$$u = \frac{1}{C} \int i \, dt$$

3.4 l'inductance :





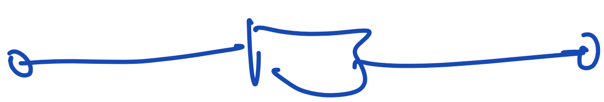
$$u = L \frac{di}{dt} \quad \begin{cases} \vec{Rot}\, \vec{H} = \vec{J} \\ \vec{Rot}\, \vec{E} = -\dfrac{d\vec{B}}{dt} \end{cases}$$

$L \; [H]$ Henry

4. Eléments de circuit :

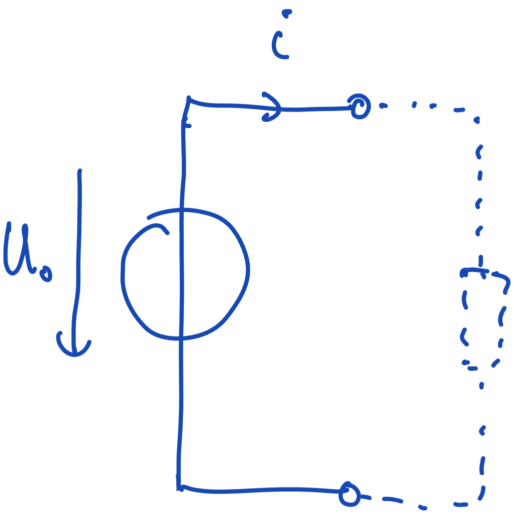
4.1 Dipôle :

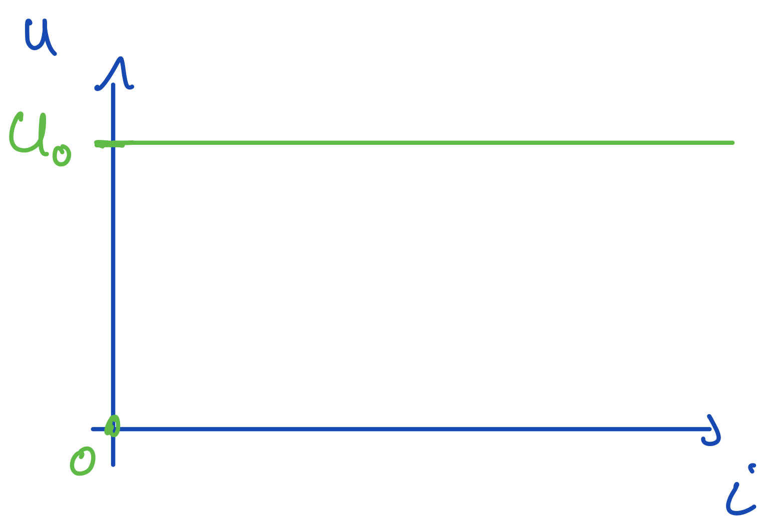
circuit qui possède 2 bornes



4.2 Sources de tension et de courant

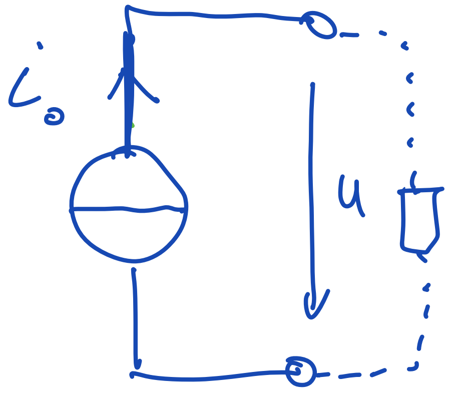
a) Source de tension idéale :

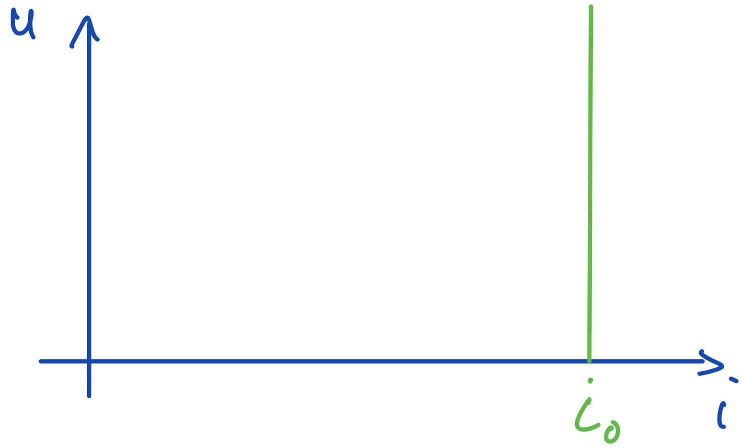




c'est un élément virtuel, idéal et inexistant dans la nature

b) Source de courant idéale :

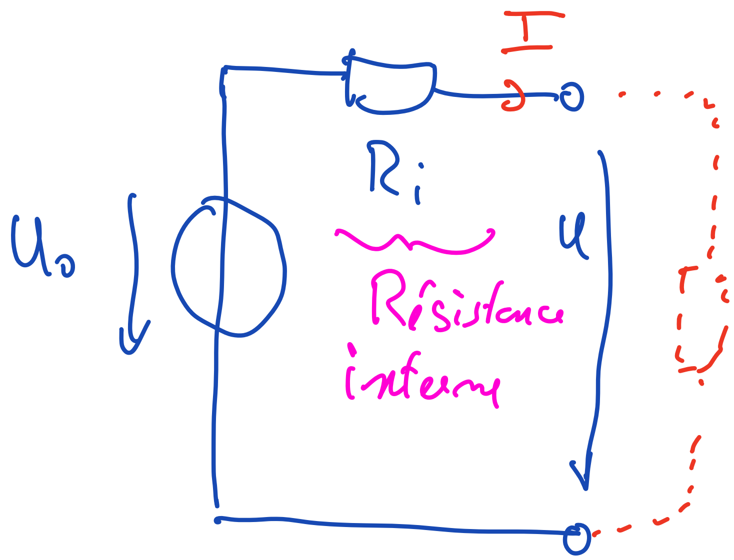




élément virtuel, inexistant dans la nature.

4.2.5 Source de tension réelle :

Déf :



S. Tension idéale

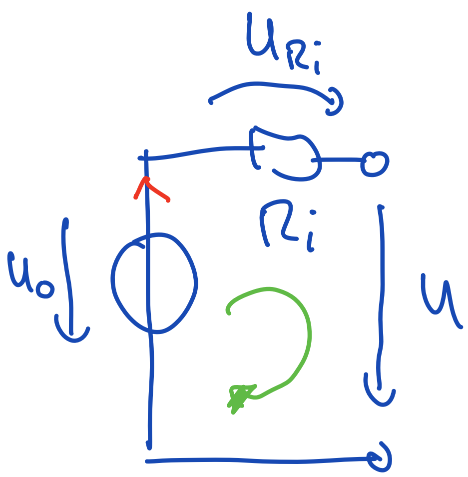
s. tension réelle

U_0 : Tension de la source idéale

Tension à vide

R_i : Résistance interne

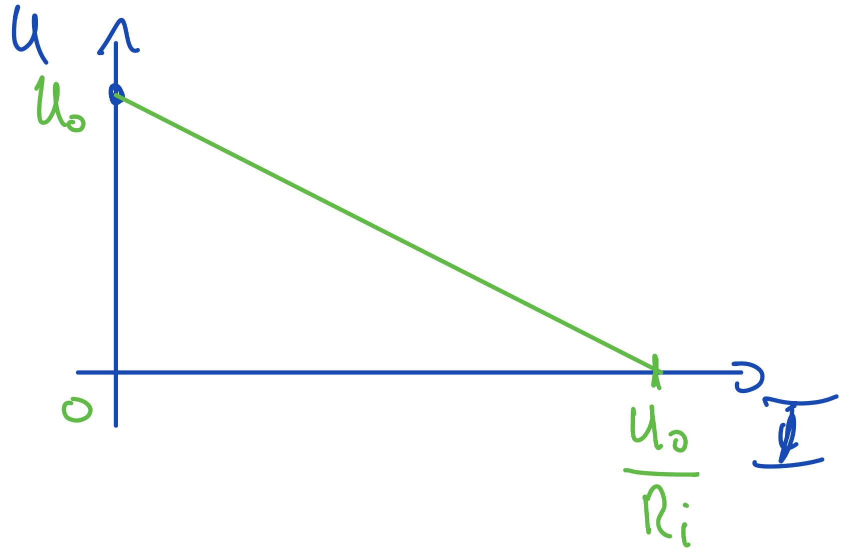
u : Tension de la source



$$\sum u = 0$$

$$-U_0 + \underbrace{U_{R_i}}_{= R_i \cdot I} + u = 0$$

$$u = U_0 - R_i \cdot I$$



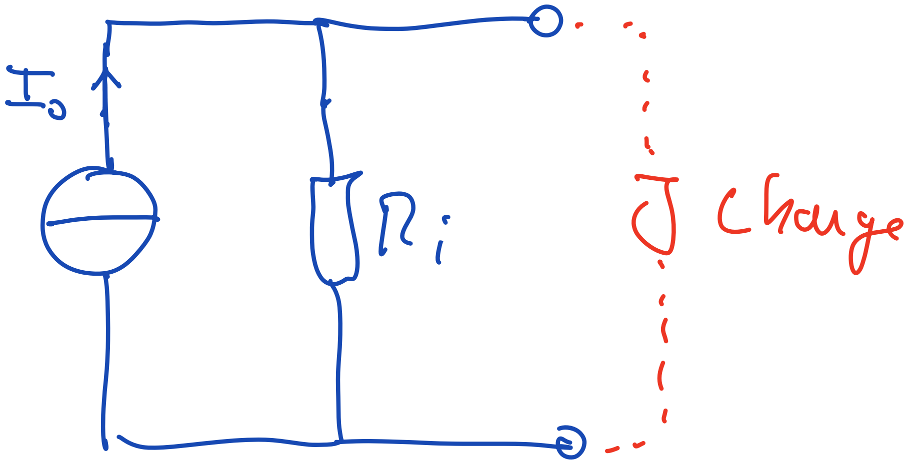
Courant max :

$$U = 0$$

$$0 = U_0 - R_i \cdot I_{cc}$$

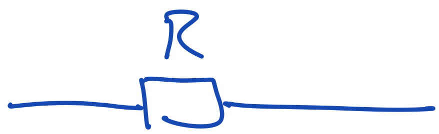
$$I_{cc} = \frac{U_0}{R_i}$$

4.2.6 Source de courant réelle :



4.3 Élément de base :

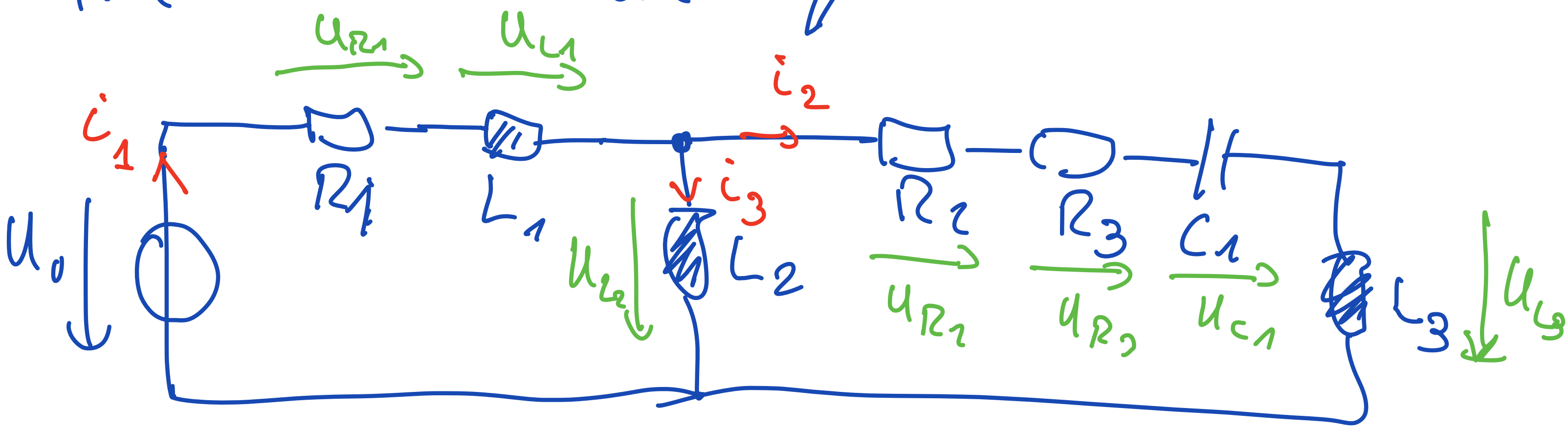
Résistance



Inductance

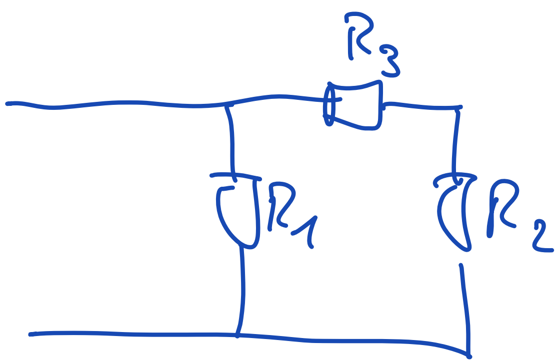
Capacité

4.4 Schéma électrique :



Recap : Quiz :

4 : // → même tension aux bornes



R_1 n'est pas en // avec R_2 !

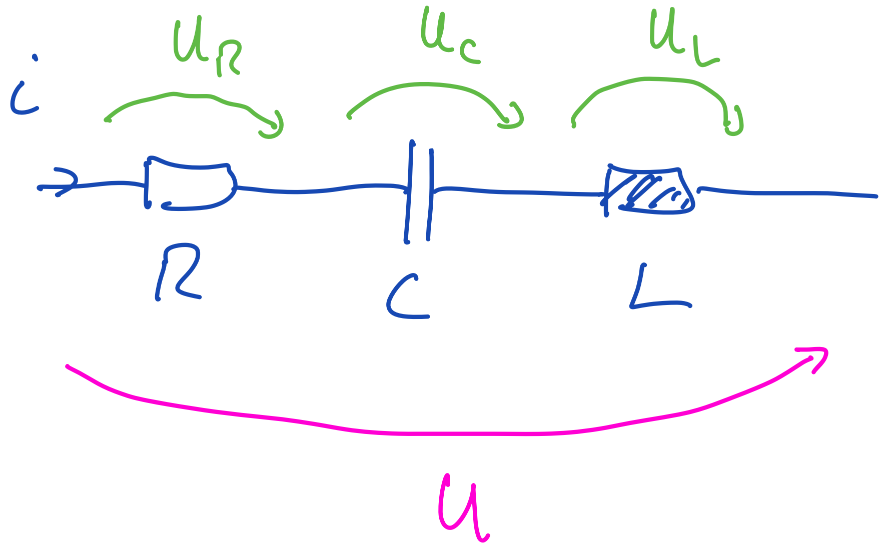
7 : Source idéale seule impossible.

8 : Source idéale est toujours constante

9 : Impossible de mettre des sources de courant en série.

5. Combinaison simple d'éléments linéaires

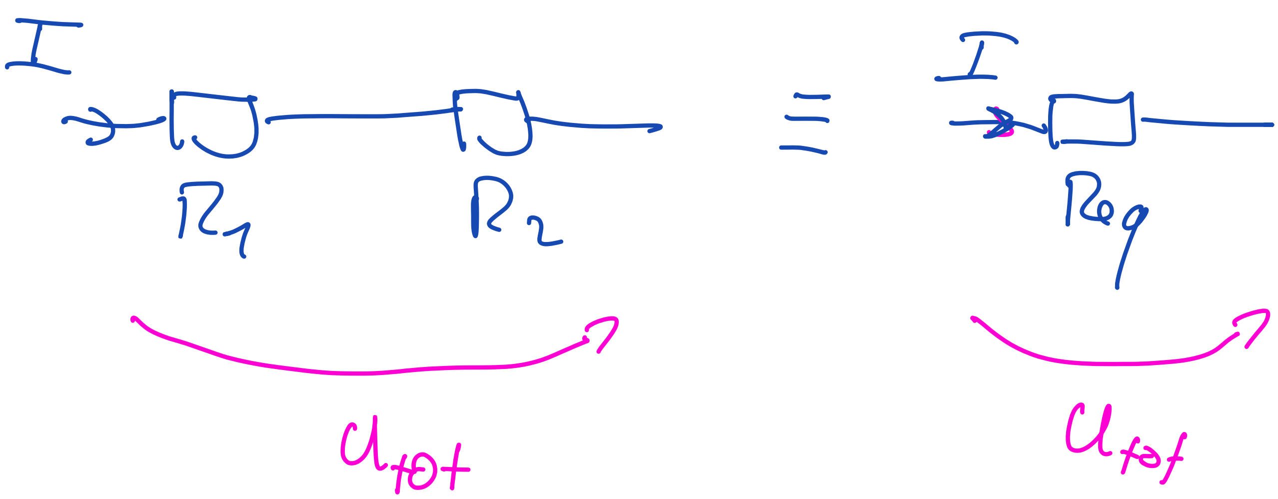
5.2 Mise en série :



Série : parcouru par le même courant

$$i_R = i_L = i_C \rightarrow \text{Série}$$

5.2.2 Mise en série de la résistance



$$U_{tot} = U_{R_1} + U_{R_2} \qquad U_{tot} = R_{eq} \cdot I$$

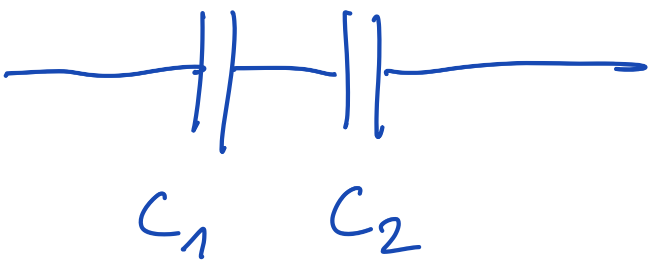
$$= R_1 I + R_2 I$$

$$= (R_1 + R_2) I = R_{eq} \cdot I$$

$$\Longrightarrow R_{eq} = R_1 + R_2$$

En Série $R_{eq} = \sum_{K=1}^{n} R_K$ (n = nb de R)

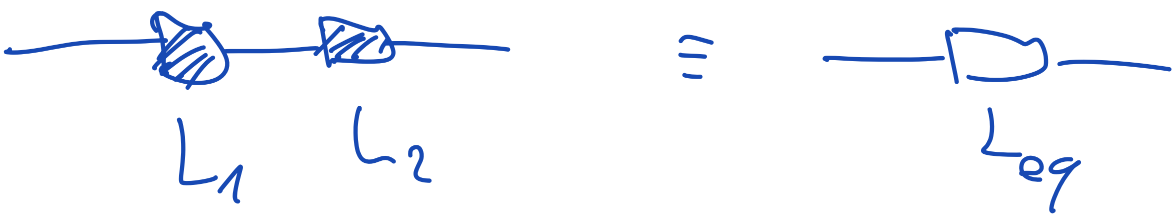
5.2.3 Mise en série des C



$\equiv$ $C_{eq} = ?$

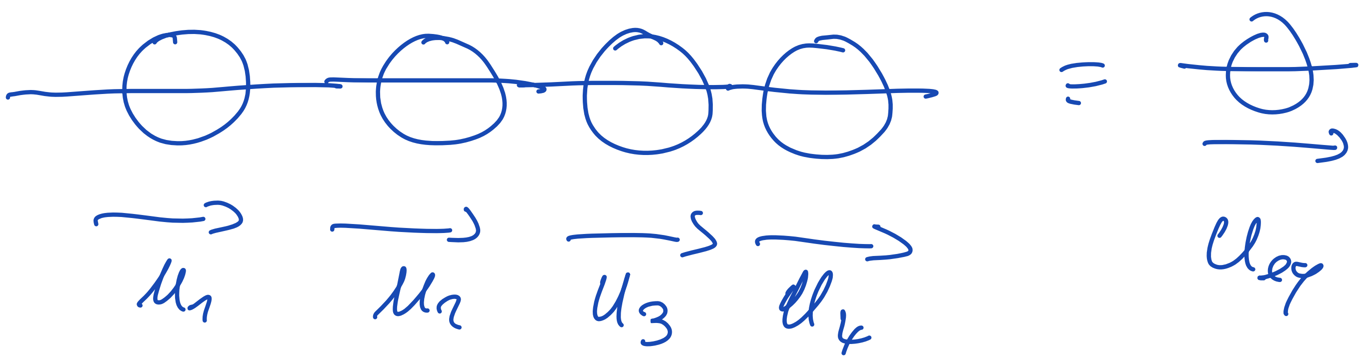
Série $C_{eq} = \dfrac{1}{\sum_{K=1}^{n} \dfrac{1}{C_K}}$ n = nb de C

5.2.6 Mise en série des L

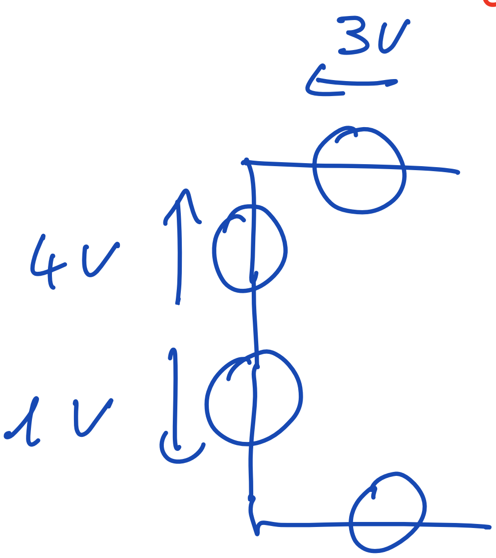


Série $L_{eq} = \sum_{k=1}^{n} L_k$ $\quad n =$ nb de L

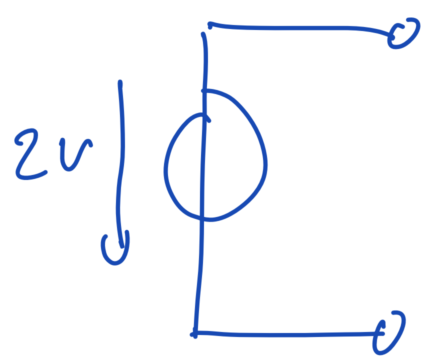
5.2.7 Mise en série de source de tension



Série $U_{eq} = \sum_{k=1}^{n} U_k$ $\quad n =$ nb de source

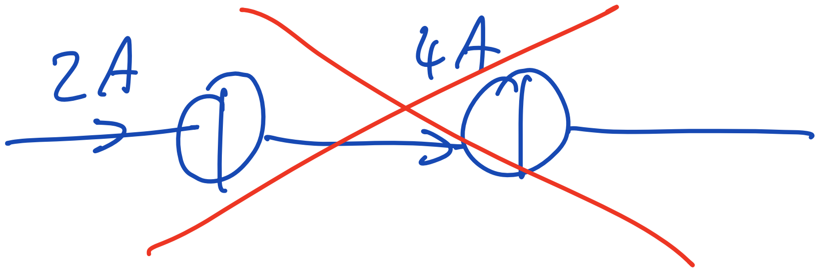


$\equiv$



$\overrightarrow{2U}$

5.2.9 Mise en série de sources de courant

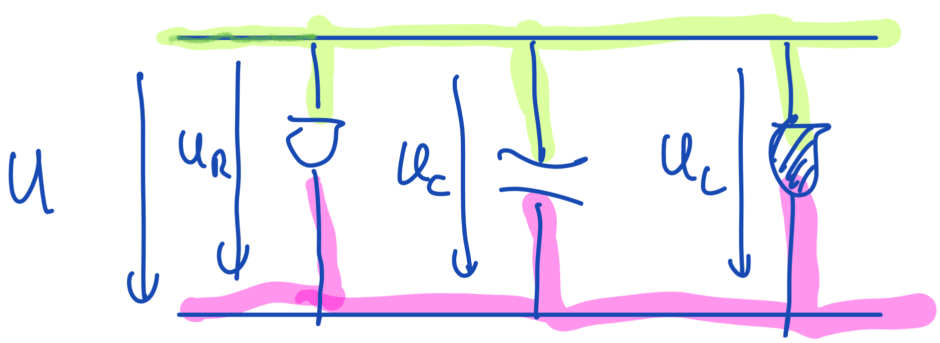


impossible

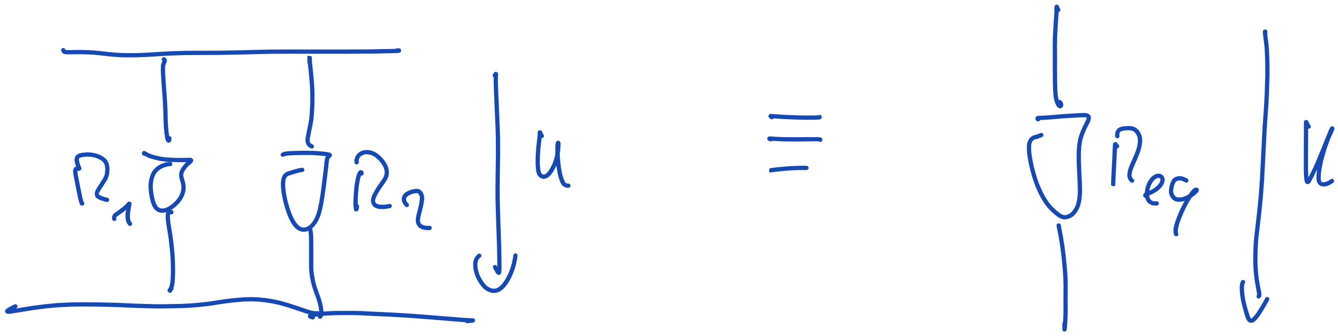
$\Rightarrow$ Impossible sauf si toutes les sources ont le même courant

5.3.2 Mise en // des R :

Définition : Toutes les bornes des éléments sont au même potentiel



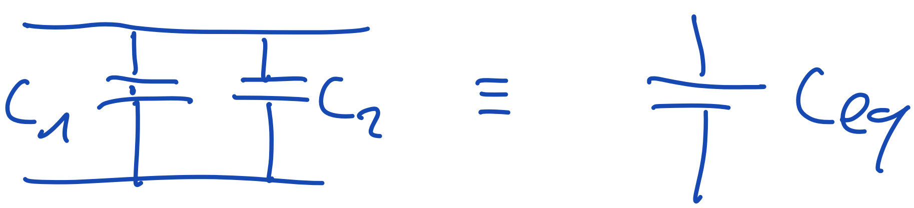
$$U_R = U_C = U_L$$



$$R_{eq} = \frac{1}{\sum_{k=1}^{m} \frac{1}{R_k}}$$ m = nb de R

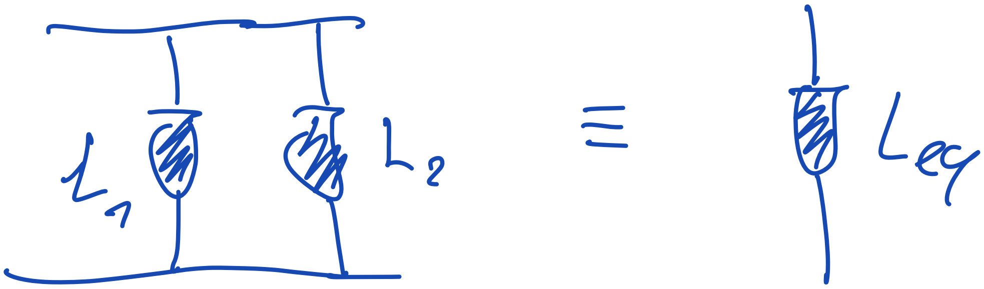
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

5.3.5 Mise en // des C :



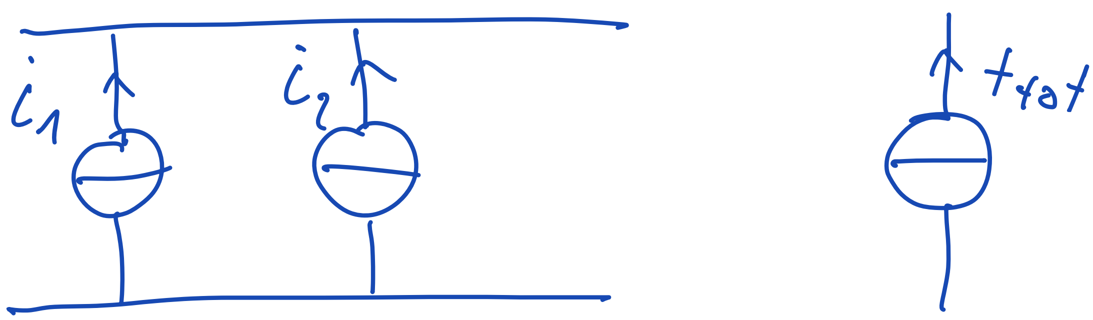
// $$C_{eq} = \sum_{k=1}^{m} C_k$$ m = nb de C

5.3.6 Mise en // des L



$$// \quad L_{eq} = \frac{1}{\sum_{k=1}^{m} \frac{1}{L_k}} \qquad m = \text{nb de } L$$

5.3.7 Mise en // des sources de courant

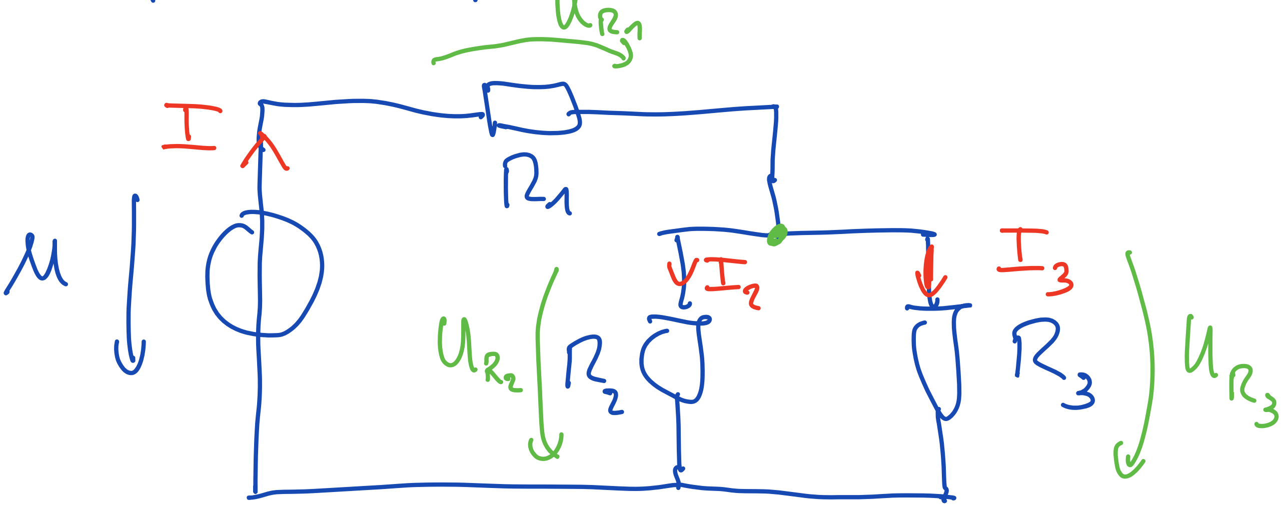


$$i_{tot} = \sum_{k=1}^{m} i_k$$

Mise en // de sources de tension

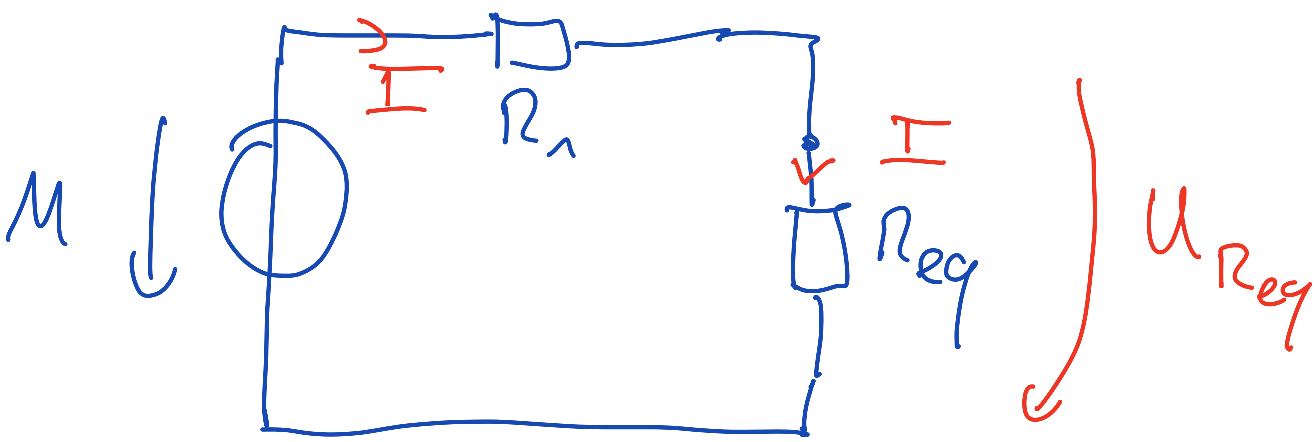
est impossible sauf si toutes les tension ont la même couleur

5.4 Circuits combinés :

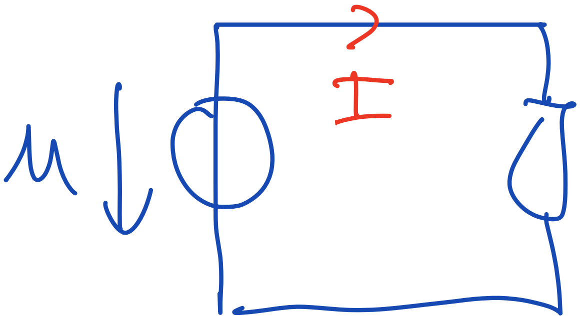


$I = ?$ $\quad I_2 ?$ $\quad I_3 ?$

$R_2 // R_3$



$$R_{eq} = \frac{R_2 \cdot R_3}{R_2 + R_3}$$



$$R_{tot} = R_1 + R_{eq} = R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}$$

$$U = R_{tot} \cdot I$$

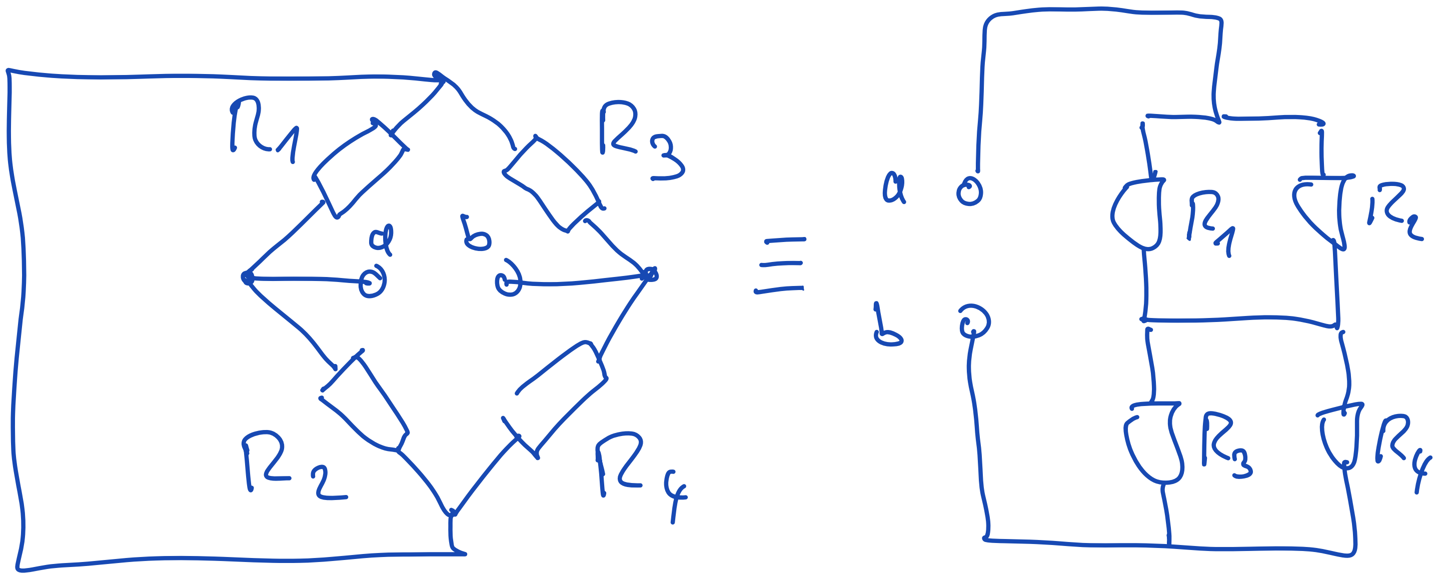
$$I = \frac{U}{R_{tot}}$$

$$U_{R_2} = U_{R_3} = U_{R_{eq}} = R_{eq} \cdot I = R_{eq} \cdot \frac{U}{R_{tot}}$$

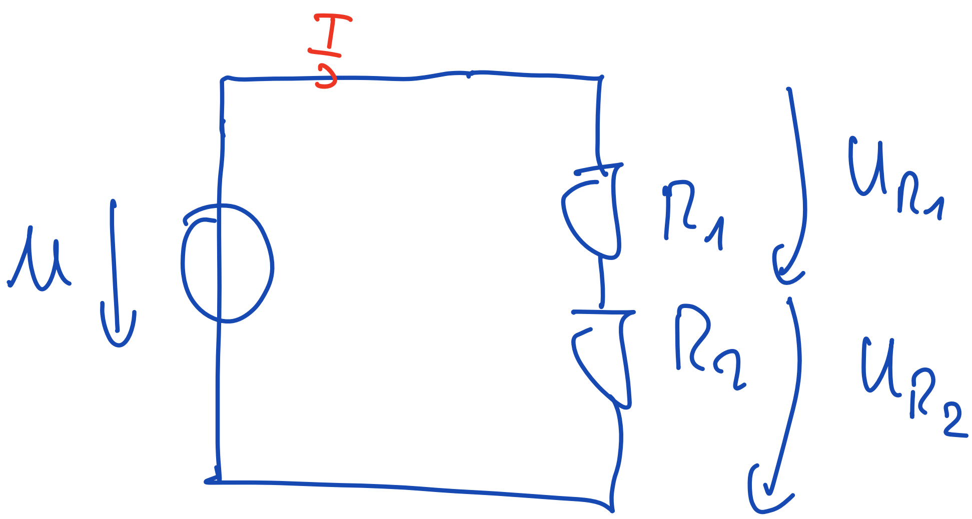
$$I_2 = \frac{U_{R_2}}{R_2} = \frac{U_{R_{eq}}}{R_2}$$

$$I_3 = \frac{U_{R_3}}{R_3} = \frac{U_{R_{eq}}}{R_3}$$

5.4.3 Exemple :



5.5.1 Diviseur de tension :

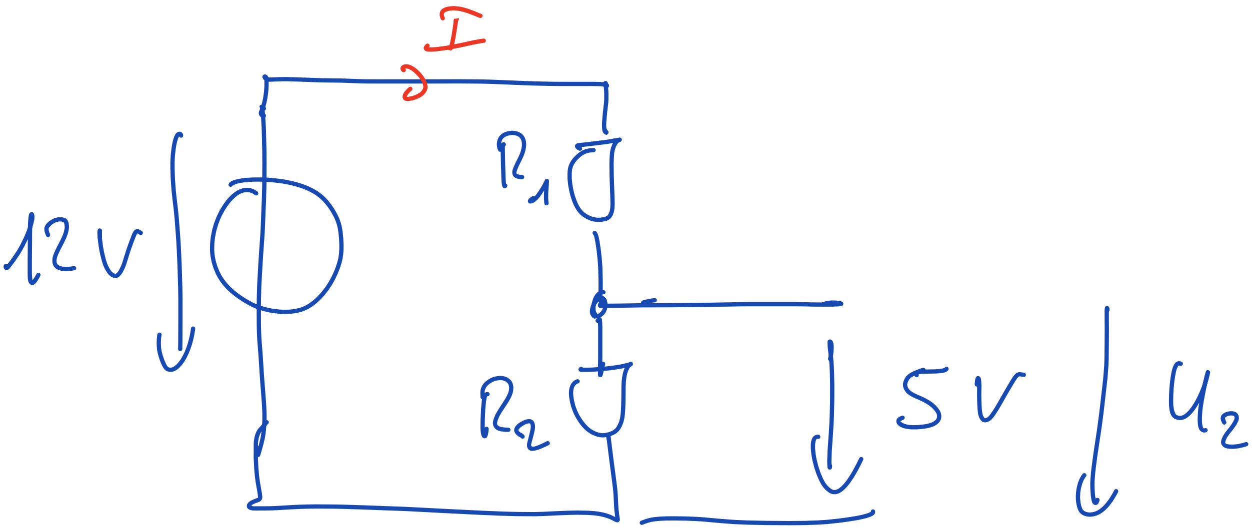


$$U = U_{R_1} + U_{R_2}$$

$$= (R_1 + R_2)\, I$$

$$I = \frac{U}{R_1 + R_2}$$

$$U_{R_2} = R_2 \cdot I = \frac{R_2}{R_1 + R_2} \cdot U$$



$$U_2 = \frac{R_1}{R_1 + R_2} \cdot U$$

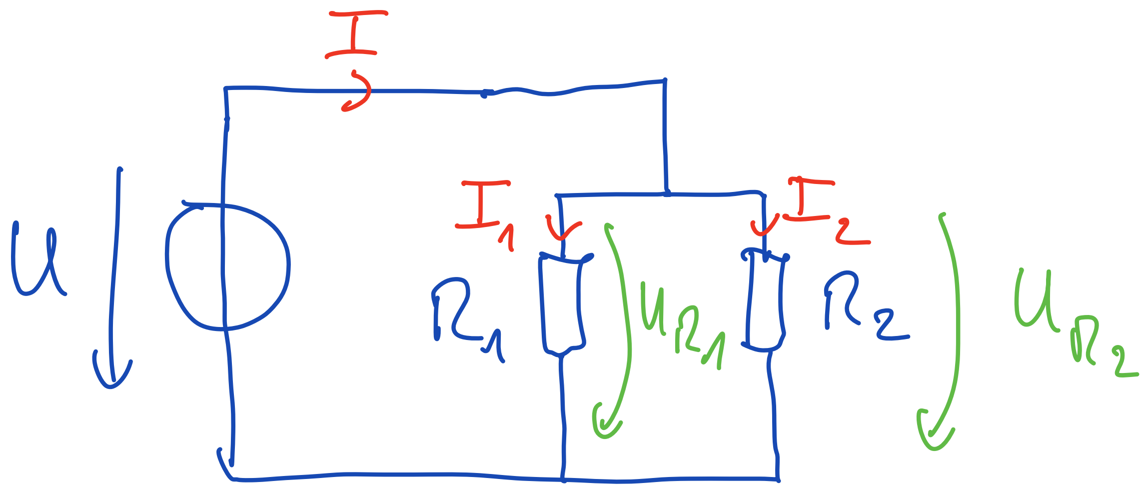
$$5 = \frac{R_2}{R_1 + R_2} \cdot 12$$

$$5 R_1 = 7 R_2$$

$R_1 = 100\,k\Omega$

$R_2 = 71{,}5\,k\Omega$

5.5.4 Diviseur de courant :



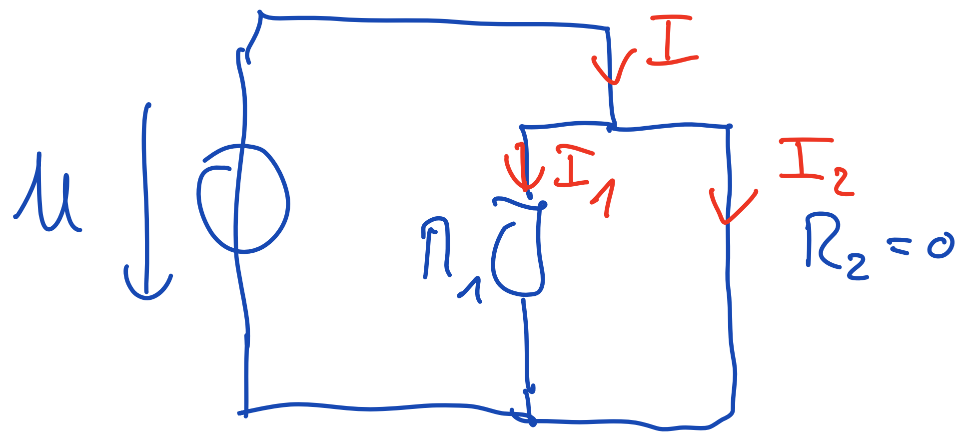
$$U = U_{R_1} = U_{R_2}$$

$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} \qquad I = \frac{U}{R_{eq}}$$

$$U_{R_2} = R_2 \cdot I_2 = U = \frac{R_1 \cdot R_2}{R_1 + R_2} \cdot I$$

$$\boxed{\begin{aligned} I_2 &= \frac{R_1}{R_1 + R_2} \cdot I \\ I_1 &= \frac{R_2}{R_1 + R_2} \cdot I \end{aligned}}$$

Si :



$$I_2 = \frac{R_1}{R_1 + \cancel{R_2}} \cdot I = I$$

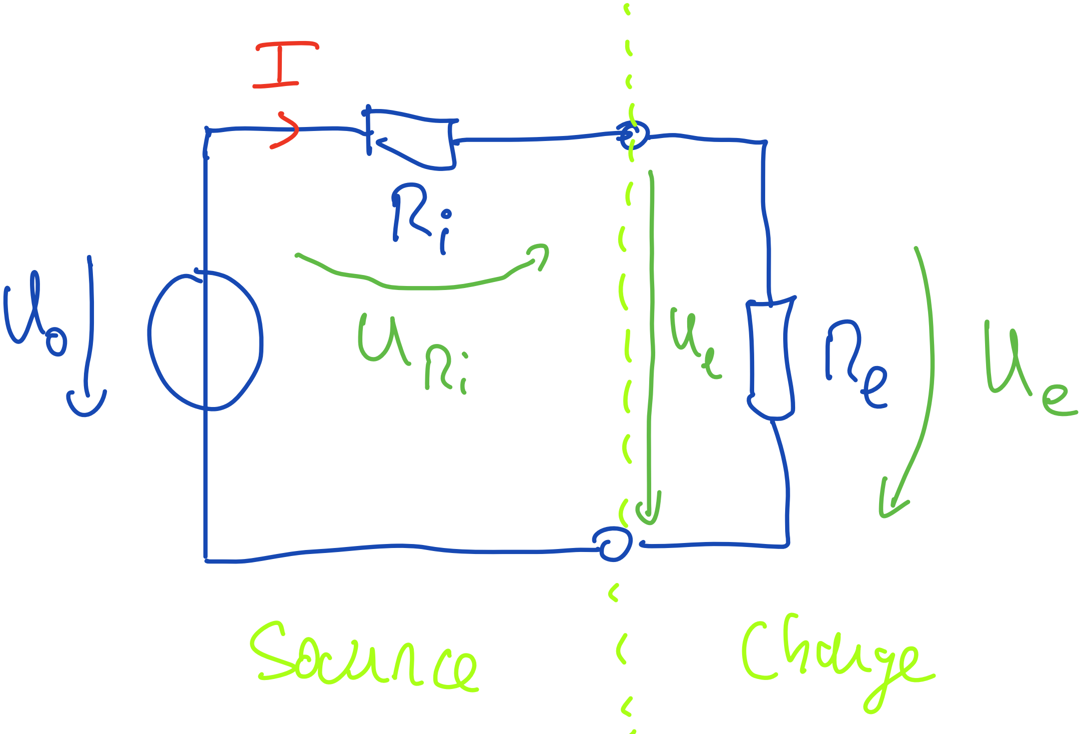
$$I_1 = 0$$

5.6 Méthodes de résolution :

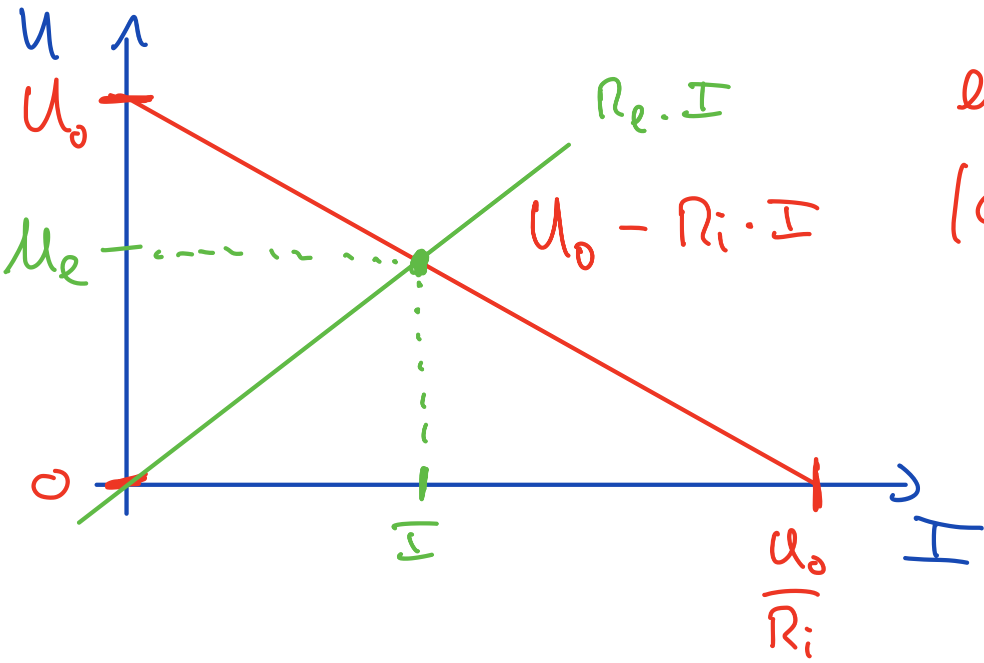
- Redessiner le schéma
- Définir toutes les grandeurs U, I, ~~?~~ → indice
- Définir le sens des flèches
- Réduire le schéma, série ou //

• Analyse !

5.6.2 Source de tension réelle :



$$U_e = U_0 - U_{R_i} = U_0 - R_i I$$



en court-circuit (cc) : $U_e = 0$

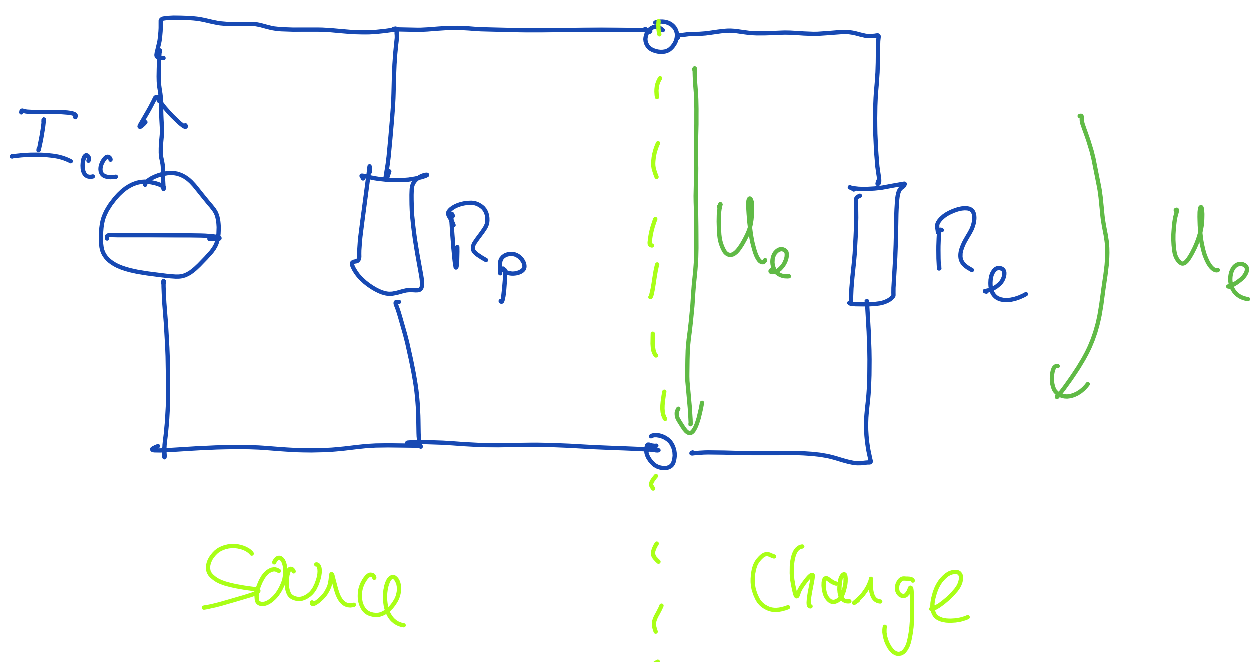
$$I_{cc} = \frac{U_0}{R_i}$$

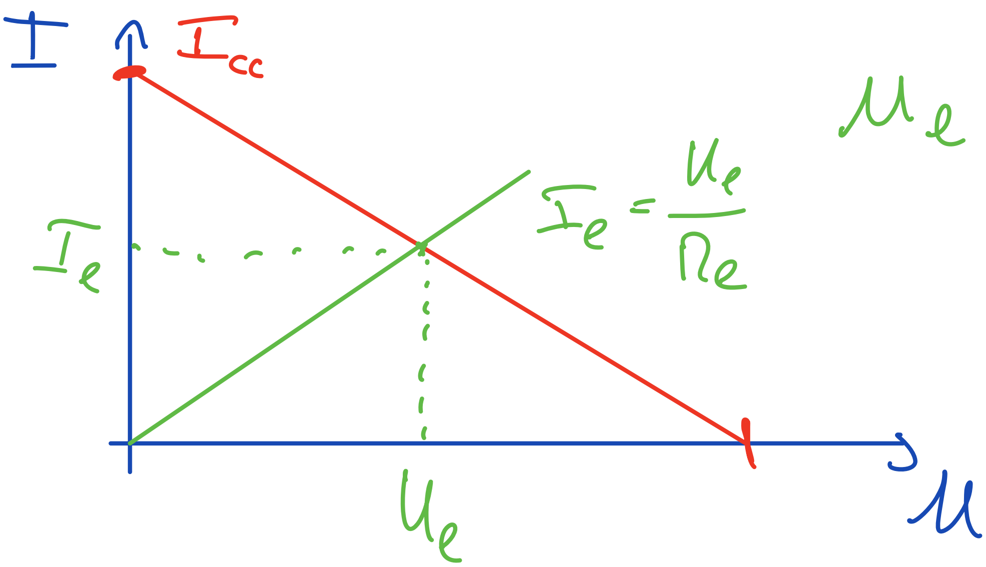
éq de la charge $U_e = R_e \cdot I$

$$U_e = U_0 \cdot \frac{R_e}{R_e + R_i}$$

$$I_e = \frac{U_0}{R_i + R_e}$$

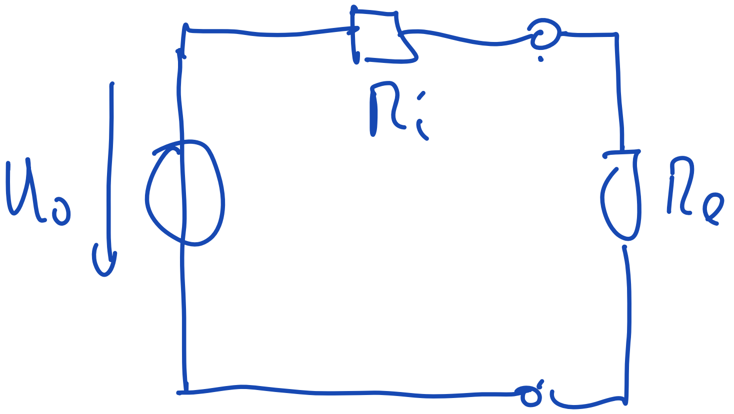
Source de courant réelle :

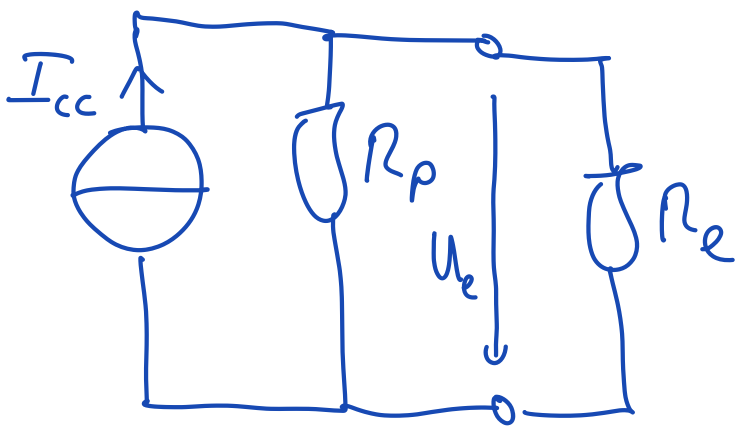




$$U_e = R_e \cdot I_e$$

5.6.3 Equivalence des sources de tension et courant réelles





court-circuit $R_e = 0$	circuit ouvert $R_e \to \infty$
$U_{e_{cc}} = 0$ $I_{e_{cc}} = \dfrac{U_0}{R_i}$	$U_{e_0} = U_0$
$I_{e_{cc}} = I_{cc}$ $U_{e_{cc}} = 0$	$U_{e_0} = R_p \cdot I_{cc}$

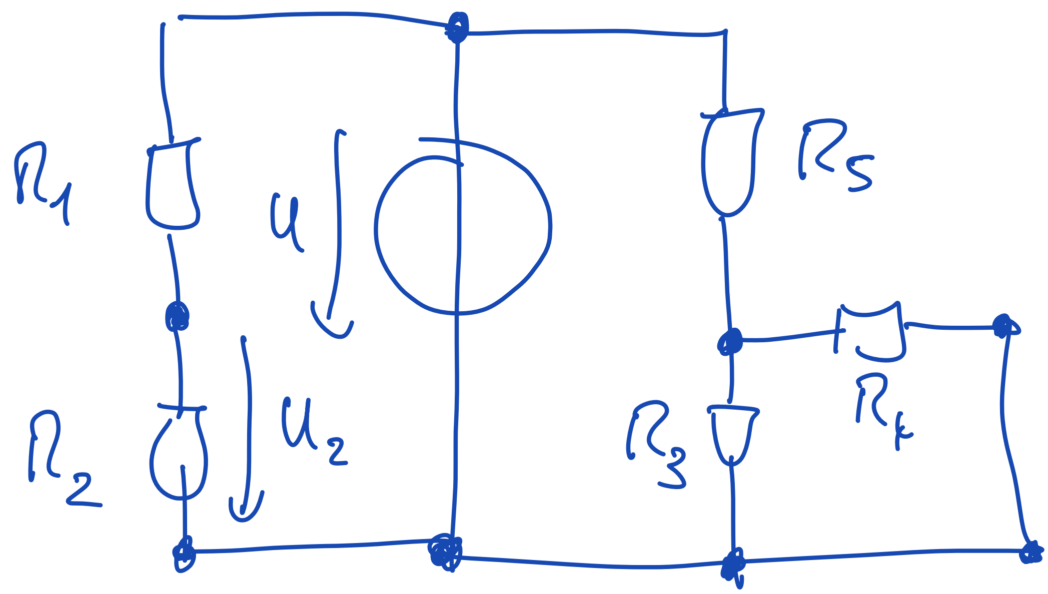
On pose :

$$U_{e_0} = R_p \cdot I_{cc} = U_0$$

$$I_{e_{cc}} = \frac{U_0}{R_i} = I_{cc}$$

$$R_p = \frac{U_0}{I_{cc}} = \frac{U_0}{U_0/R_i} = R_i$$

Quiz :



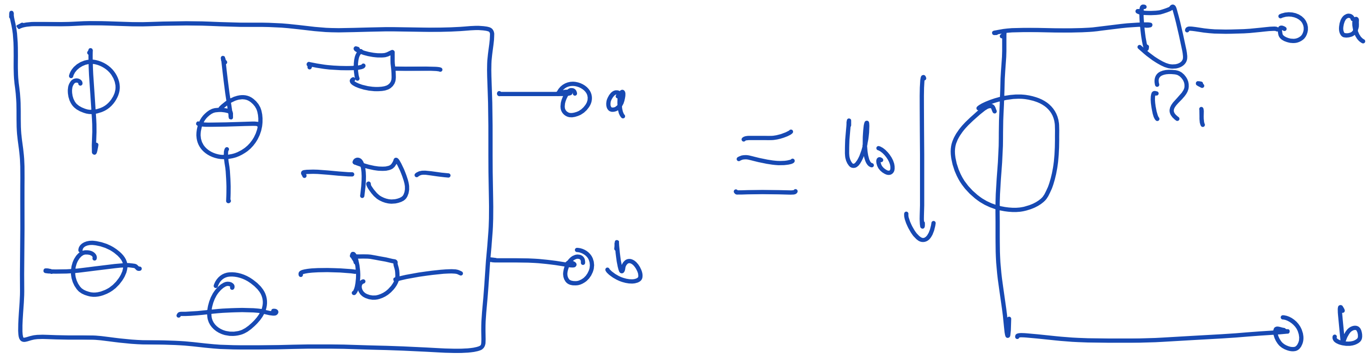
$U = 12\,V$

$$U_2 = U \cdot \frac{R_2}{R_1+R_2} = 5\,V$$

En résumé :

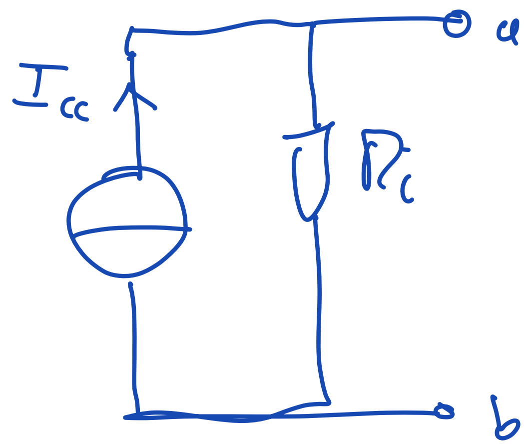
$$I_{cc} = \frac{U_0}{R_i}$$

5.7 Théorèmes de Thévenin et Norton :



U_0 = Tension à vide

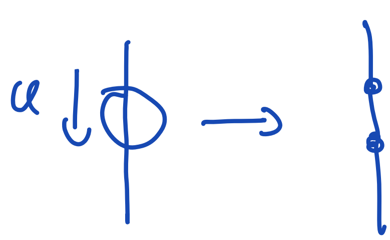
$$U_0 = U_{ab}\Big|_{I_{ab}=0} \quad \text{(à vide)} \qquad \equiv$$

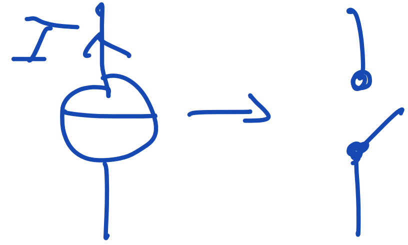


$$I_{cc} = I_{ab}\Big|_{U_{ab}=0} \quad \text{(en court-circuit)}$$

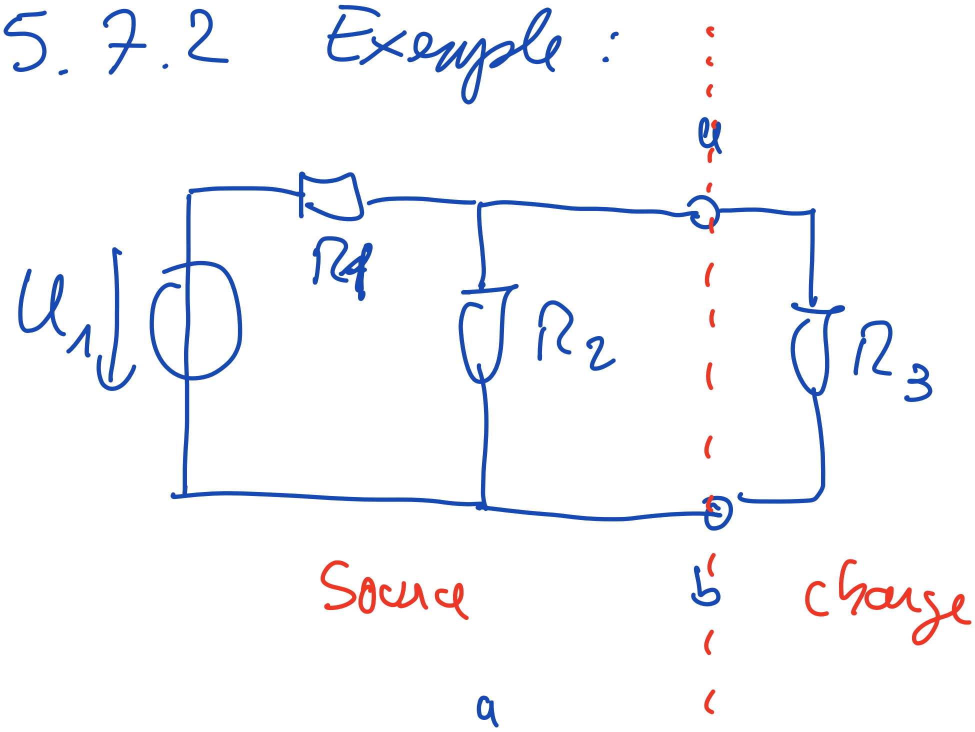
$$R_i = \frac{U_0}{I_{cc}} = R_{ab}\Big|_{\substack{U_j = 0 \\ I_j = 0}}$$

Annuler une source $\Rightarrow$

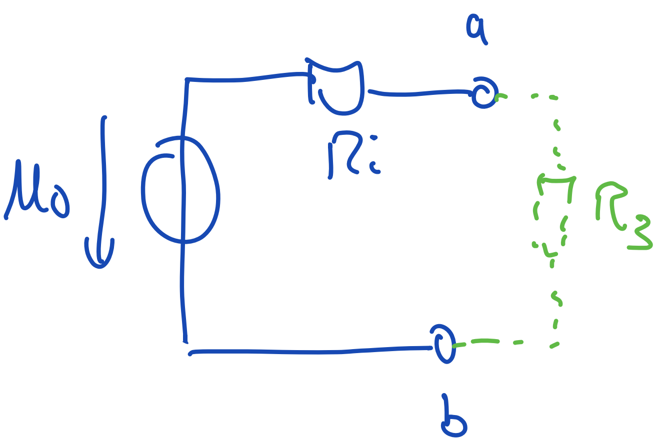




5.7.2 Exemple :

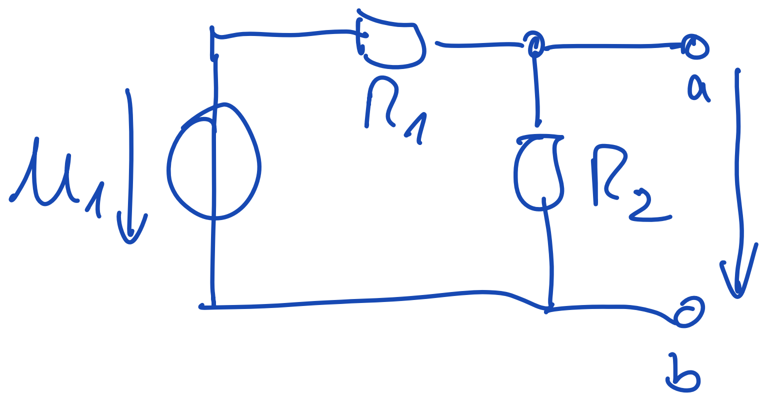


$U_3 = f(R_3)$



U_0 ? Tension à vide entre a et b ?

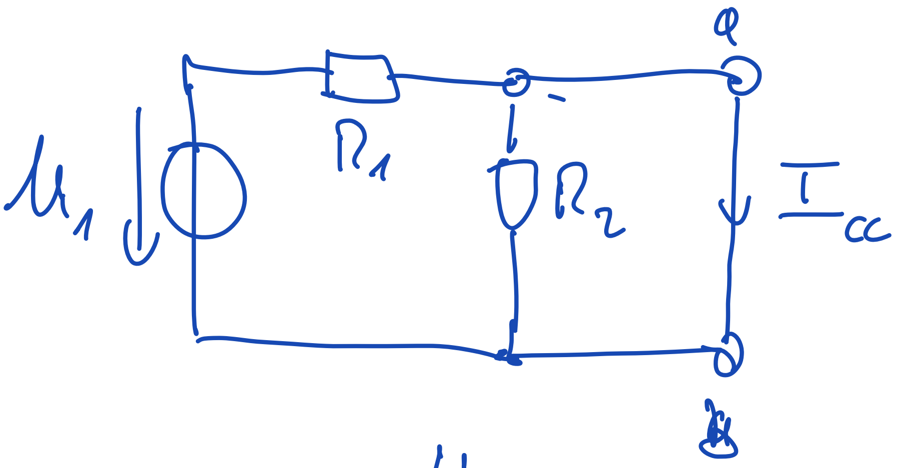
a)



$U_{R_2} = U_0$

$$U_{R_2} = U_0 = U_1 \cdot \frac{R_2}{R_1 + R_2}$$

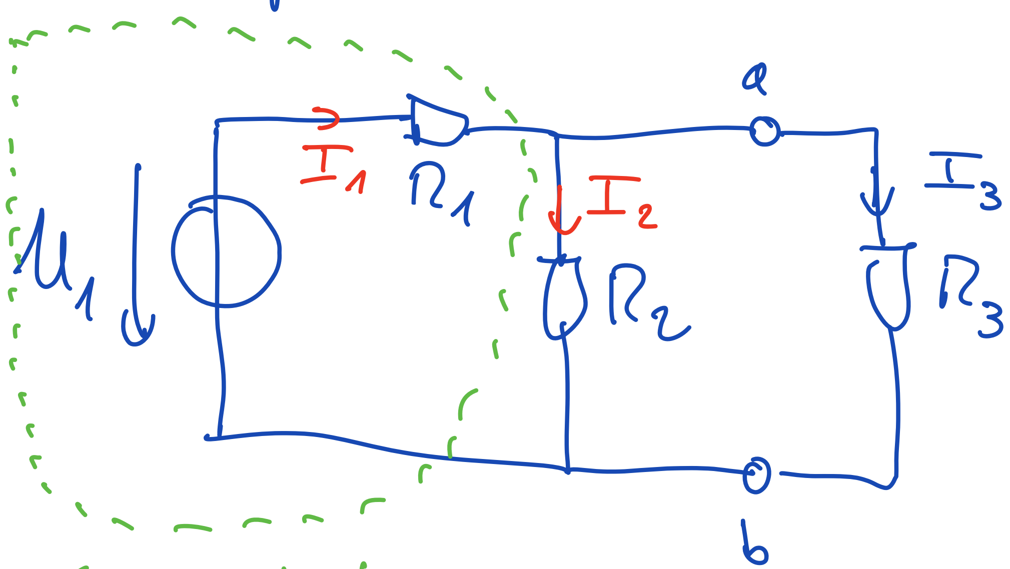
b) I_{cc} = courant de court circuit



$$I_{cc} = \frac{U_1}{R_1}$$

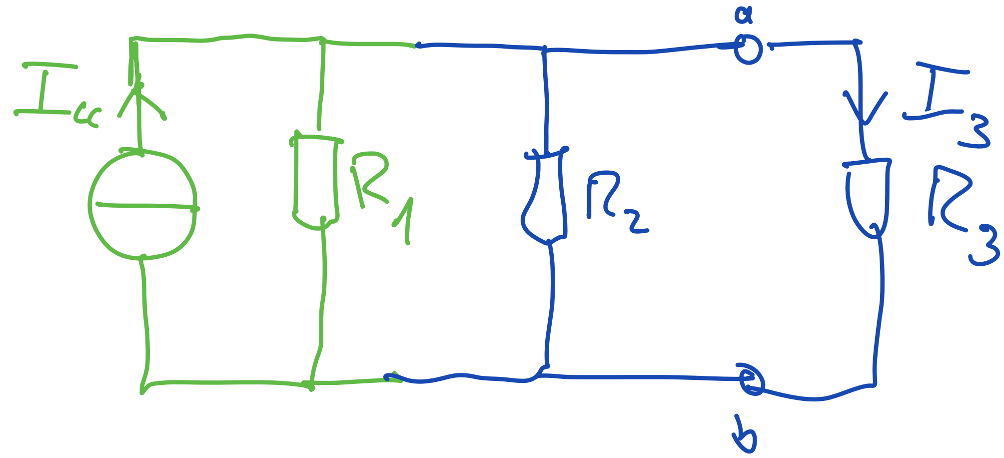
c) $$R_i = \frac{U_0}{I_{cc}} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Autre possibilité :

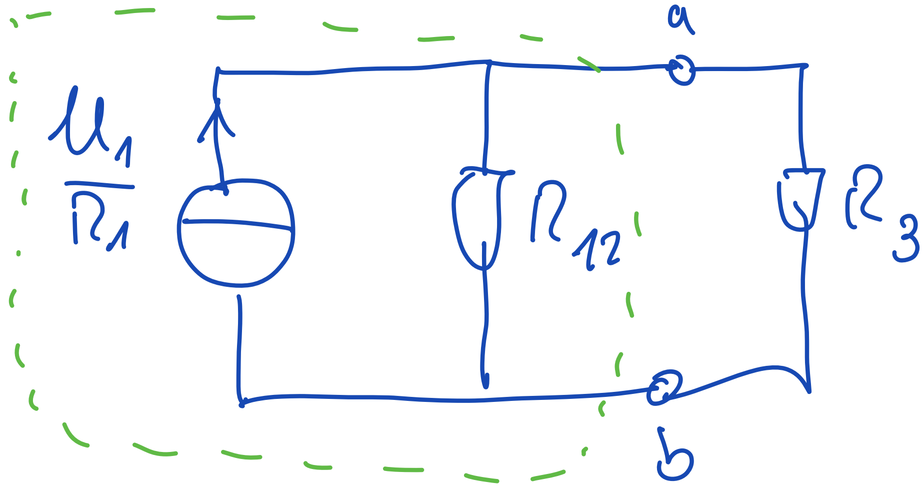


U_3 ?

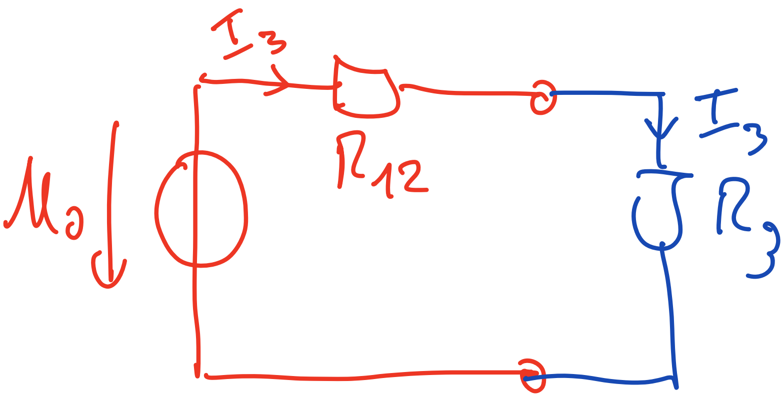
Source de tension réelle



$$I_{cc} = \frac{U_1}{R_1} \qquad R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$



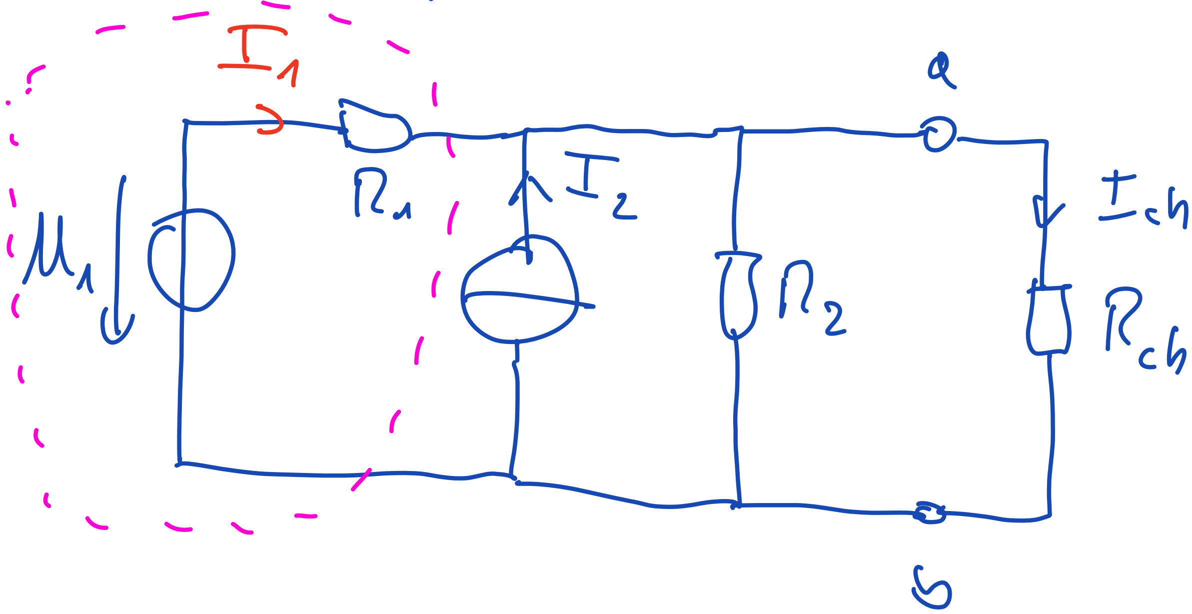
Source de courant réelle

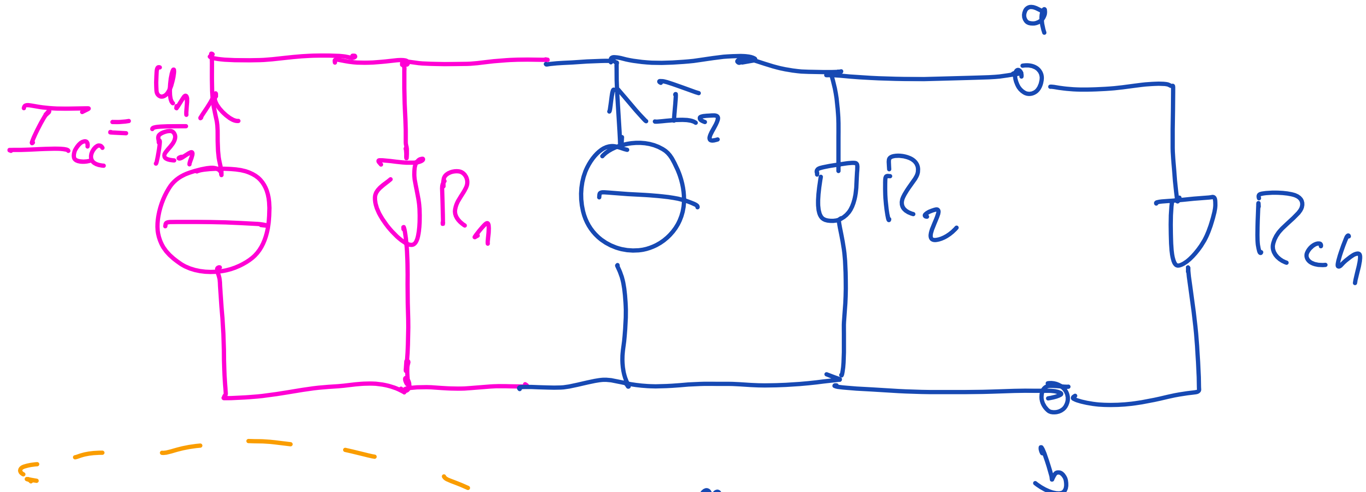


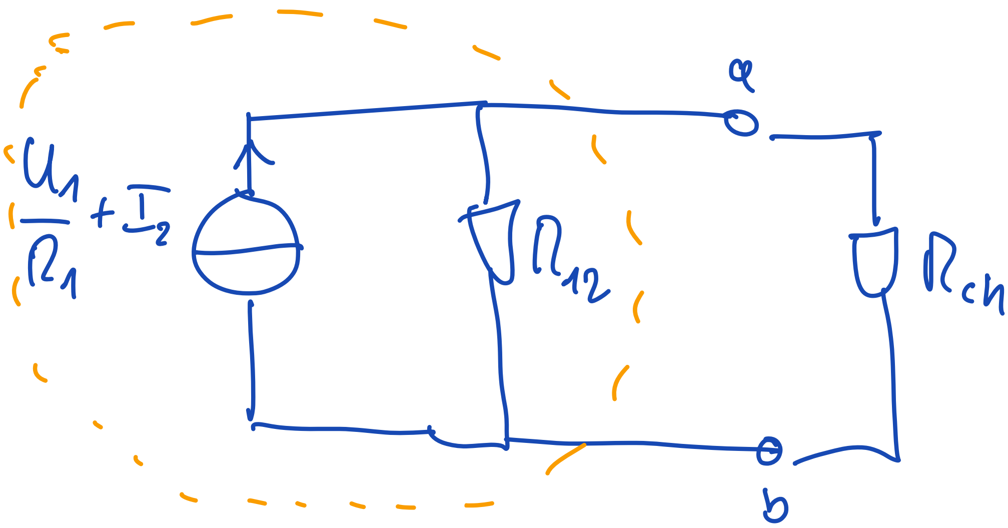
$$U_0 = R_{12} \cdot \frac{U_1}{R_1}$$

$$I_3 = \frac{U_0}{R_{12} + R_3}$$

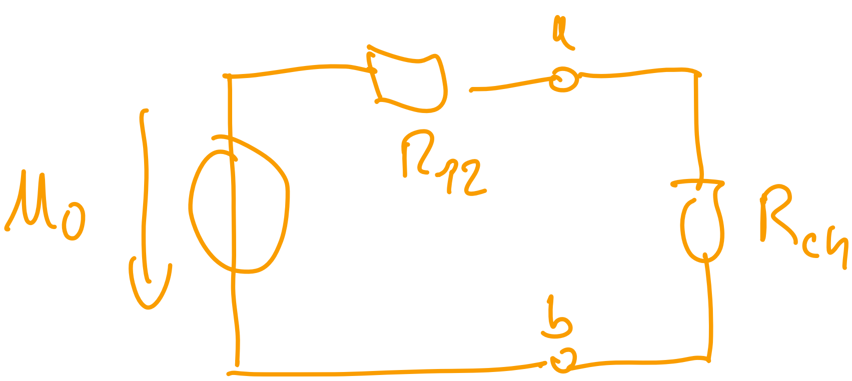
Autre exemple :







$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

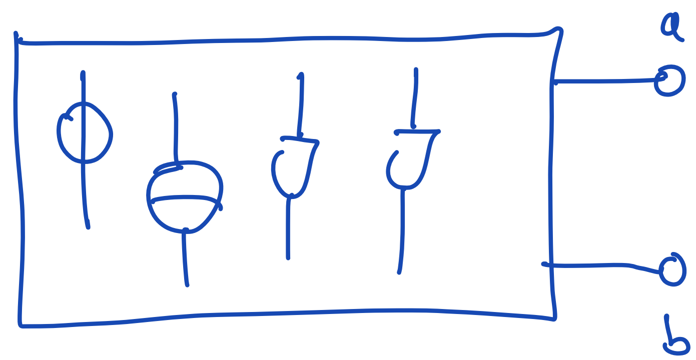


$$U_0 = R_{12}\left(\frac{U_1}{R_1} + I_2\right)$$

$$I_{ch} = \frac{U_0}{R_{12} + R_{ch}}$$

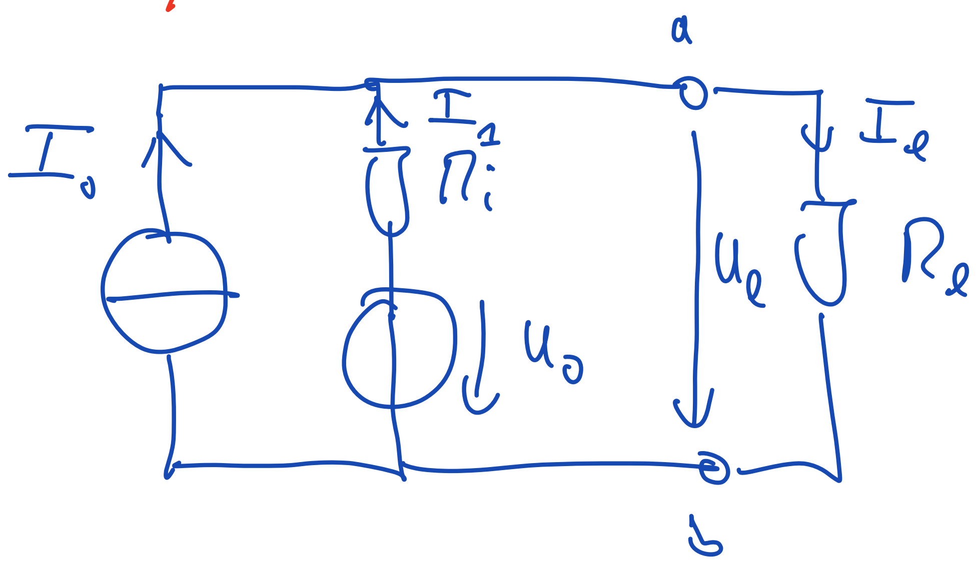
5.8 Principe de superposition

Définition :



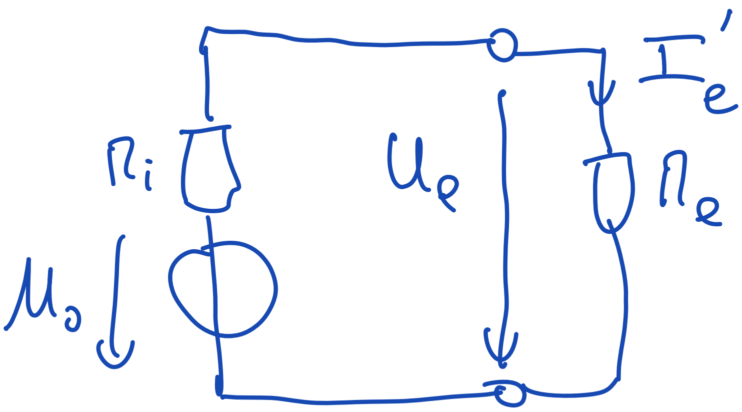
L'action résultante est la somme algébrique des actions séparées de chaque source, les autres étant annulées

Le système doit être linéaire !!



I_e, U_e ?

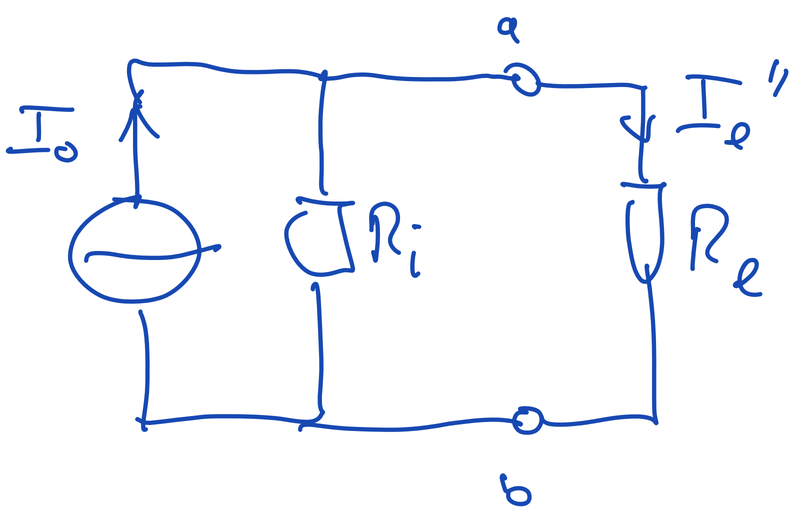
1) On annule la source de courant :



$$I_e' = \frac{U_0}{R_i + R_e}$$

$$U_e' = U_0 \cdot \frac{R_e}{R_e + R_i}$$

2) On annule la source de tension :



$$I_e'' = I_0 \cdot \frac{R_i}{R_i + R_e}$$

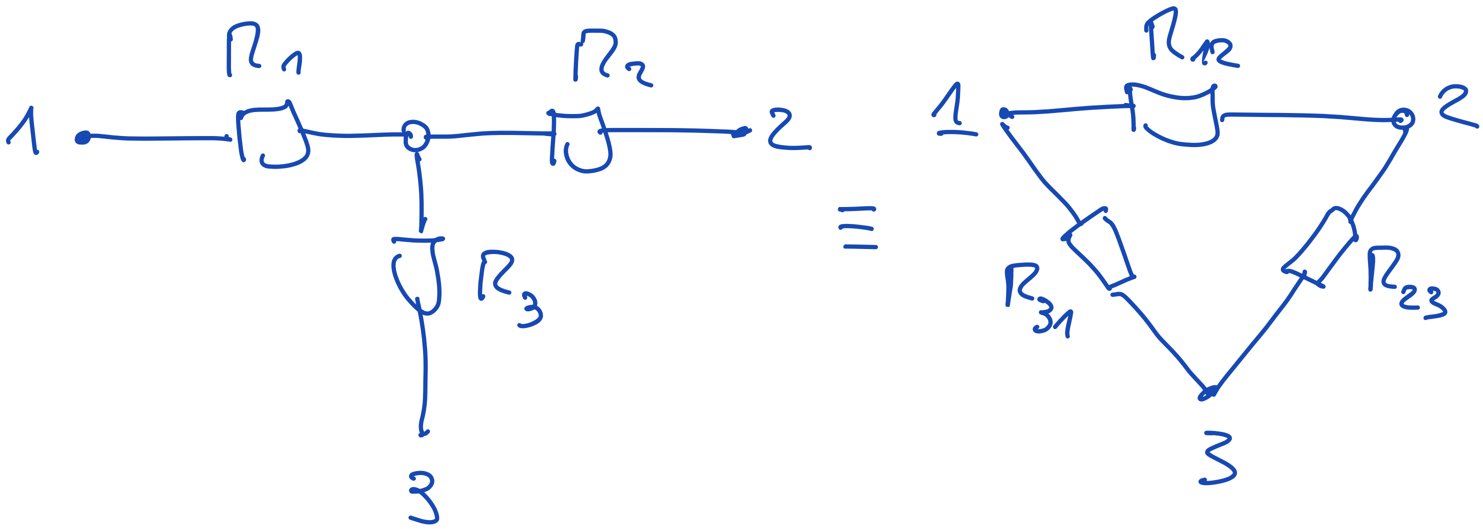
$$U_e'' = R_e \cdot I_e'' = I_0 \cdot \frac{R_e \cdot R_i}{R_e + R_i}$$

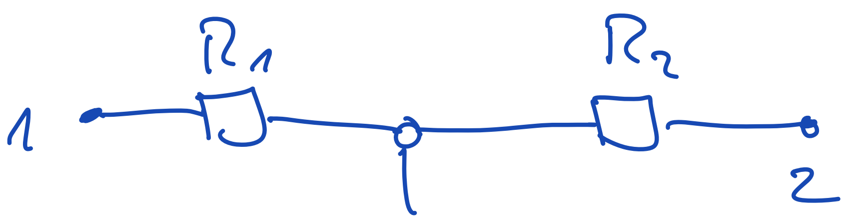
$$I_e = I_e' + I_e''$$

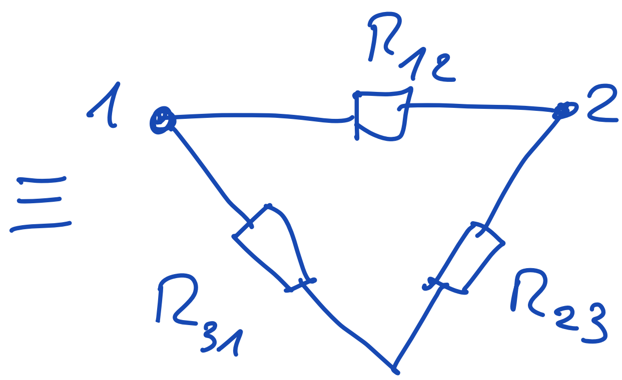
$$u_e = u_e' + u_e''$$

5.9 Transformation Etoile - Triangle

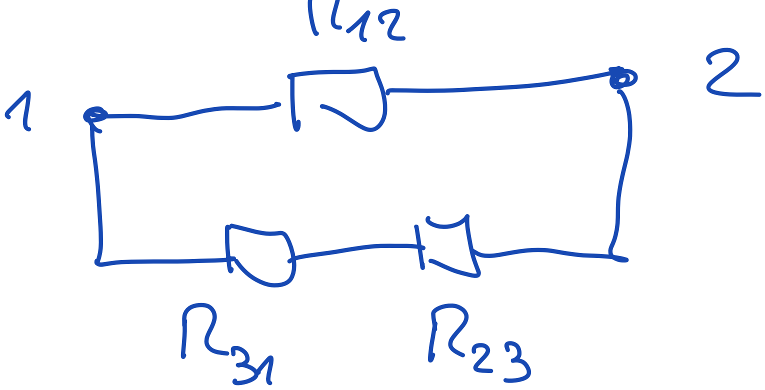
il s'agit d'un tripole :







$$R_{eq} = R_1 + R_2$$



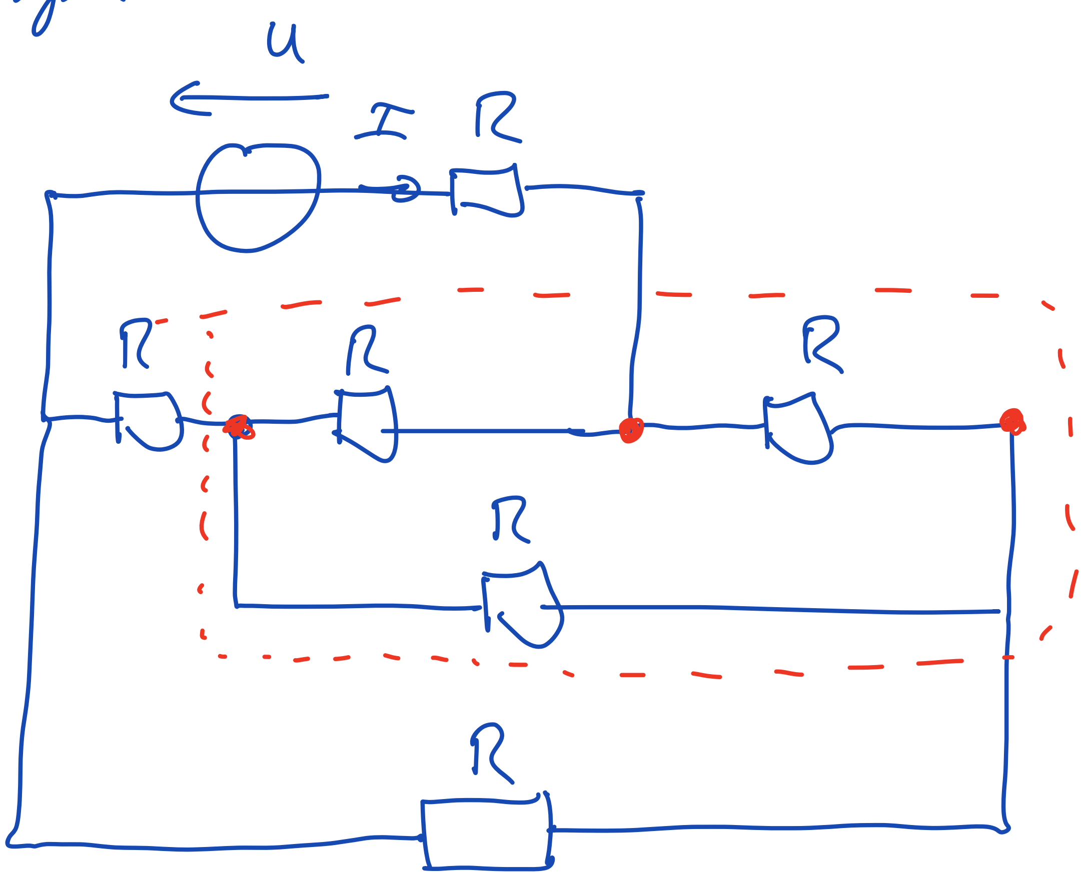
$$R_{eq} = \frac{R_{12} \cdot (R_{31} + R_{23})}{R_{12} + R_{31} + R_{23}}$$

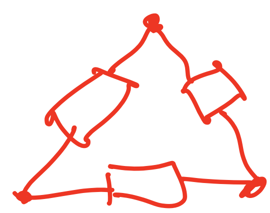
$$R_1 = \frac{R_{31} \cdot R_{12}}{R_{12} + R_{23} + R_{31}} \qquad R_{12} = R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}$$

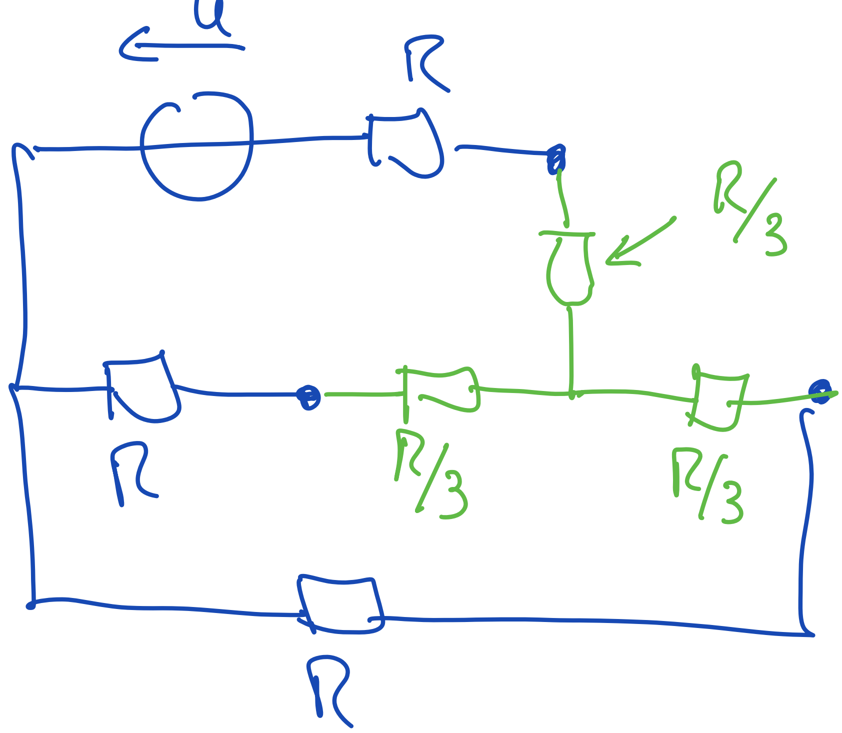
$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}} \qquad R_{23} = R_2 + R_3 + \frac{R_2 \cdot R_3}{R_1}$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}} \qquad R_{31} = R_3 + R_1 + \frac{R_3 \cdot R_1}{R_2}$$

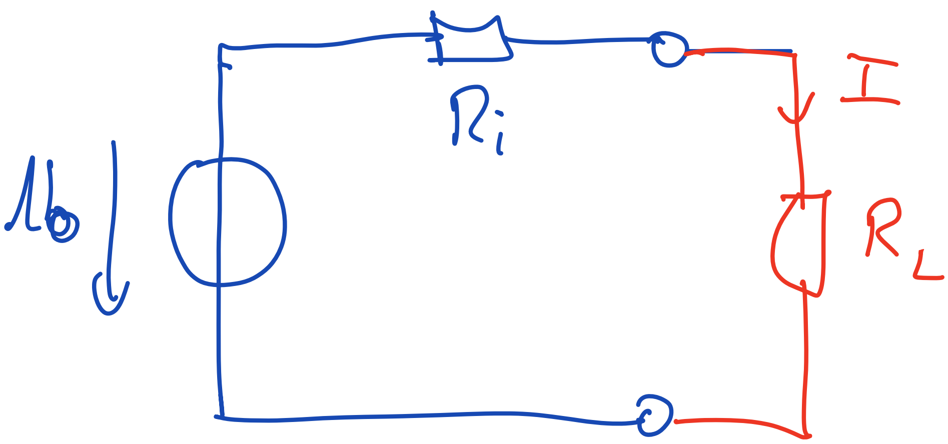
Exemple :







5.11 Puissance maximum transmise par un dipôle :



Puissance transmise à $R_L \to$ max

$$P_{R_L} = R_L \cdot I^2 = U_{R_L} \cdot I$$

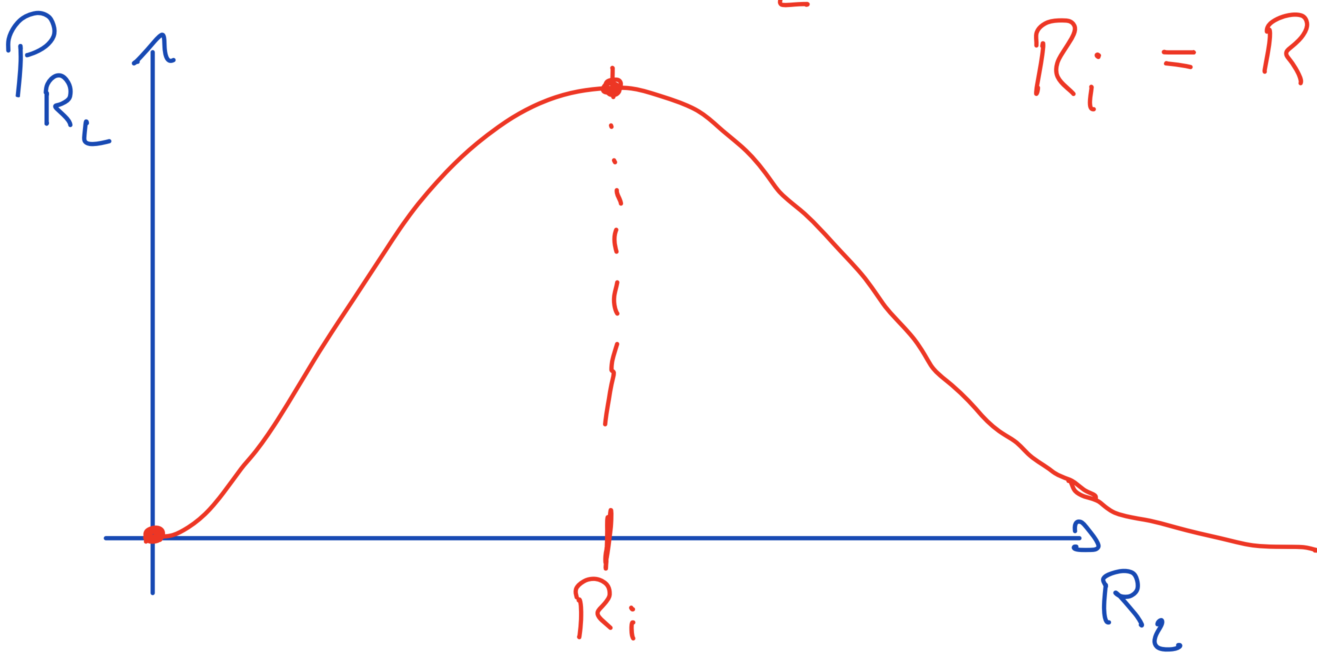
$$I = \frac{U_0}{R_i + R_L}$$

$$P_{R_L} = R_L \frac{U_0^2}{(R_i + R_L)^2}$$

Qui doit valoir R_L par rapport à R_i pour P_{R_L} max

Max : $\rightarrow$ $$\frac{d P_{R_L}}{d R_L} = 0$$

$$R_i = R_L$$



$\rightarrow$ l'adaptation de puissance

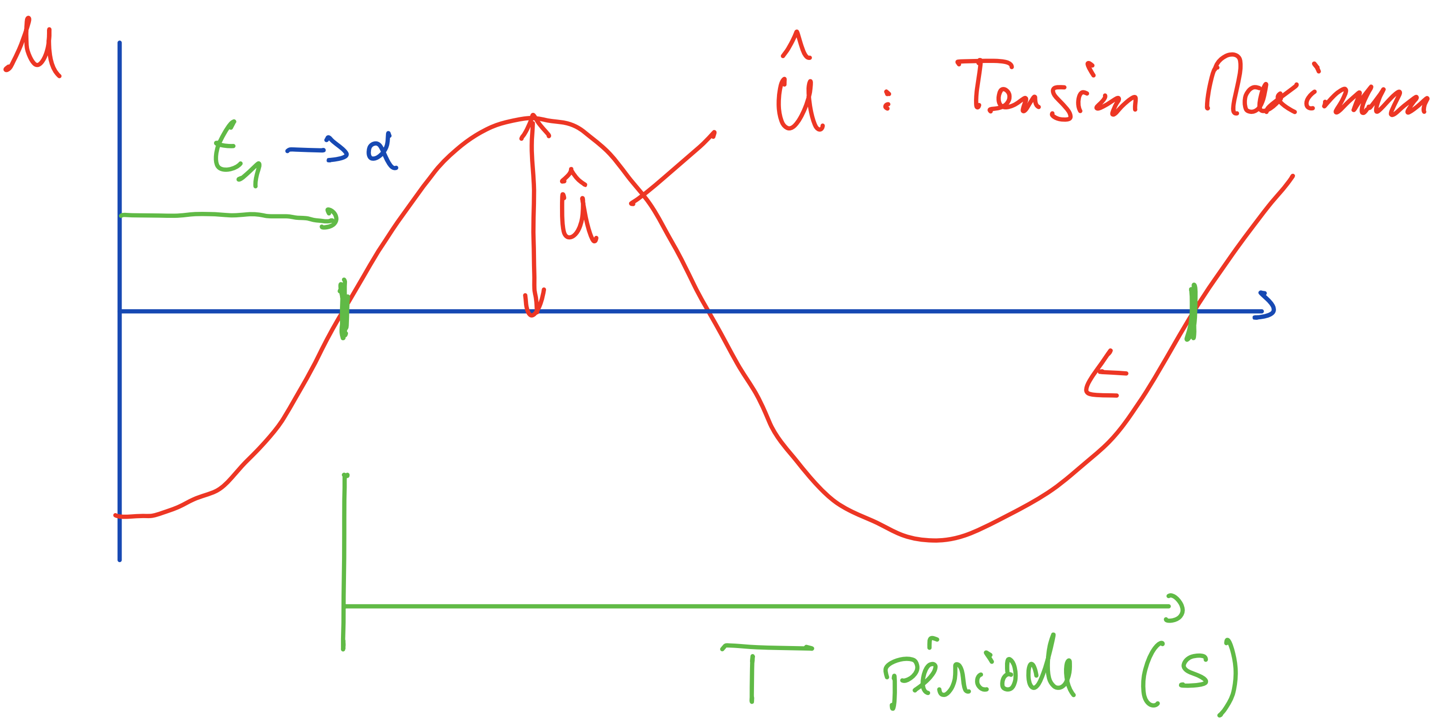
$$\eta = \frac{P_{utile}}{P_{consommée}} = \frac{R_L \cdot \cancel{I^2}}{(R_L + R_i)\cancel{I^2}}$$

Si $R_i = R_L$

$\Rightarrow \eta = 0,5$

6. Régime sinusoïdal monophasé :

6.2 Grandeurs sinusoïdales



T = période du signal (s)

f = fréquence $= \dfrac{1}{T}$ (Hz)

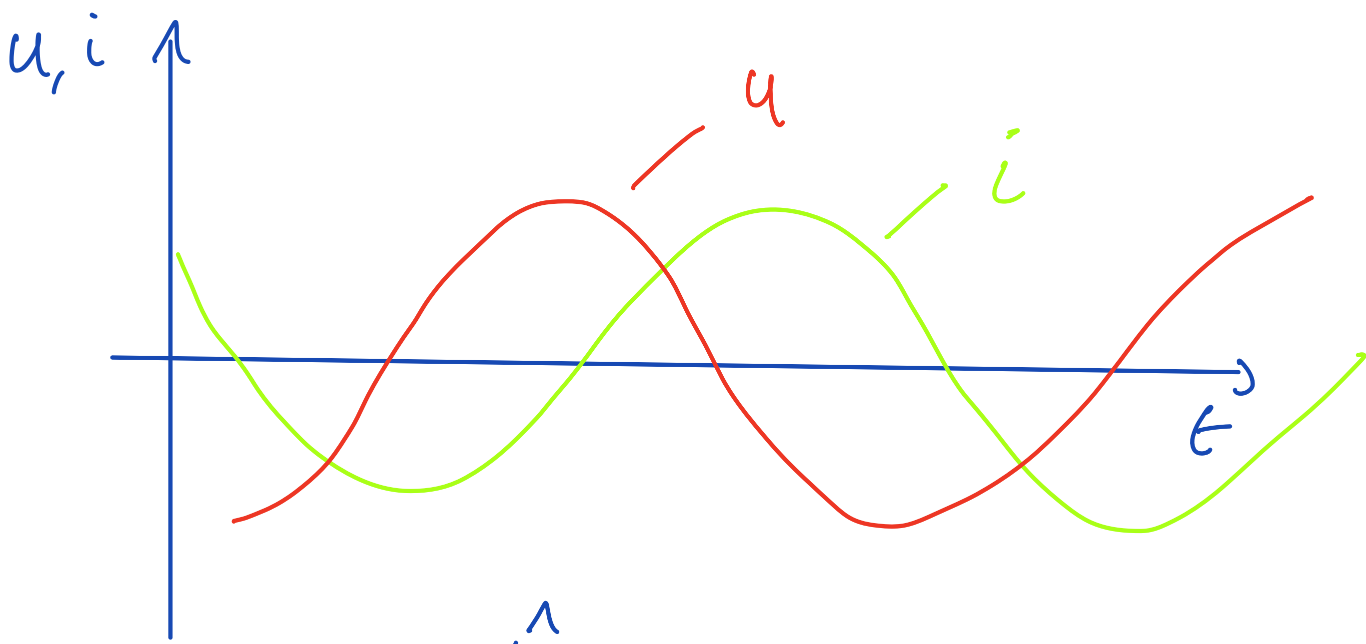
ω = pulsation $= \dfrac{2\pi}{T} = 2\pi \cdot f$

$$\alpha = \frac{t_1 \cdot 2\pi}{T}$$

$$u(t) = \hat{U} \sin(\omega t + \alpha)$$

$$u = \hat{U} \sin(\omega t + \alpha$$

minuscule : → grandeur instantanée



$$u = \hat{U} \sin(\omega t + \alpha)$$

$$i = \hat{I} \sin(\omega t + \beta)$$

Définition : $\varphi = \alpha - \beta$

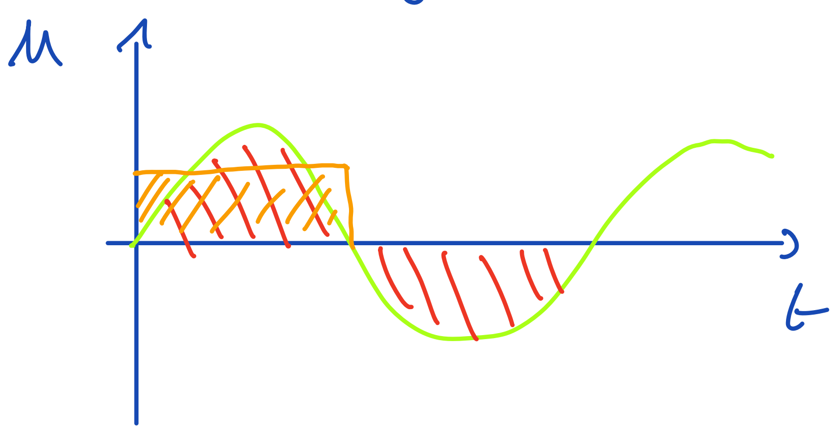
déphasage entre u et i

Définition de la valeur moyenne :

$$\bar{x} = \frac{1}{T}\int_0^T x(t)\,dt$$

$$\bar{u} = \frac{1}{T}\int_0^T u\,dt$$

$$\bar{u} = \frac{1}{T}\int_0^T \hat{U}\sin(\omega t+\alpha)\,dt$$



Moyenne sur $T/2$:

$T/2$

$$\bar{u}\Big|_{T/2} = \frac{1}{T/2}\int_0^{T/2} \hat{u}\sin(\omega t)\,dt \qquad \alpha = 0$$

$$\bar{u}\Big|_{T/2} = \frac{1}{\omega}\,\frac{2\hat{u}}{T}\left[-\cos\left(\omega\cdot\frac{T}{2}\right) + \cos(0)\right]$$

$$\omega = \frac{2\pi}{T}$$

$$\frac{\cancel{2}\,\hat{u}\,\cancel{T}}{\cancel{T}\,\cancel{2}\cdot\pi}\left[-\cos\left(\frac{\cancel{T}}{\cancel{2}}\,\frac{\cancel{2}\pi}{\cancel{T}}\right) + \cos(0)\right]$$

$$= \frac{\hat{u}}{\pi}\left[1 + 1\right] = \frac{2}{\pi}\hat{u}$$

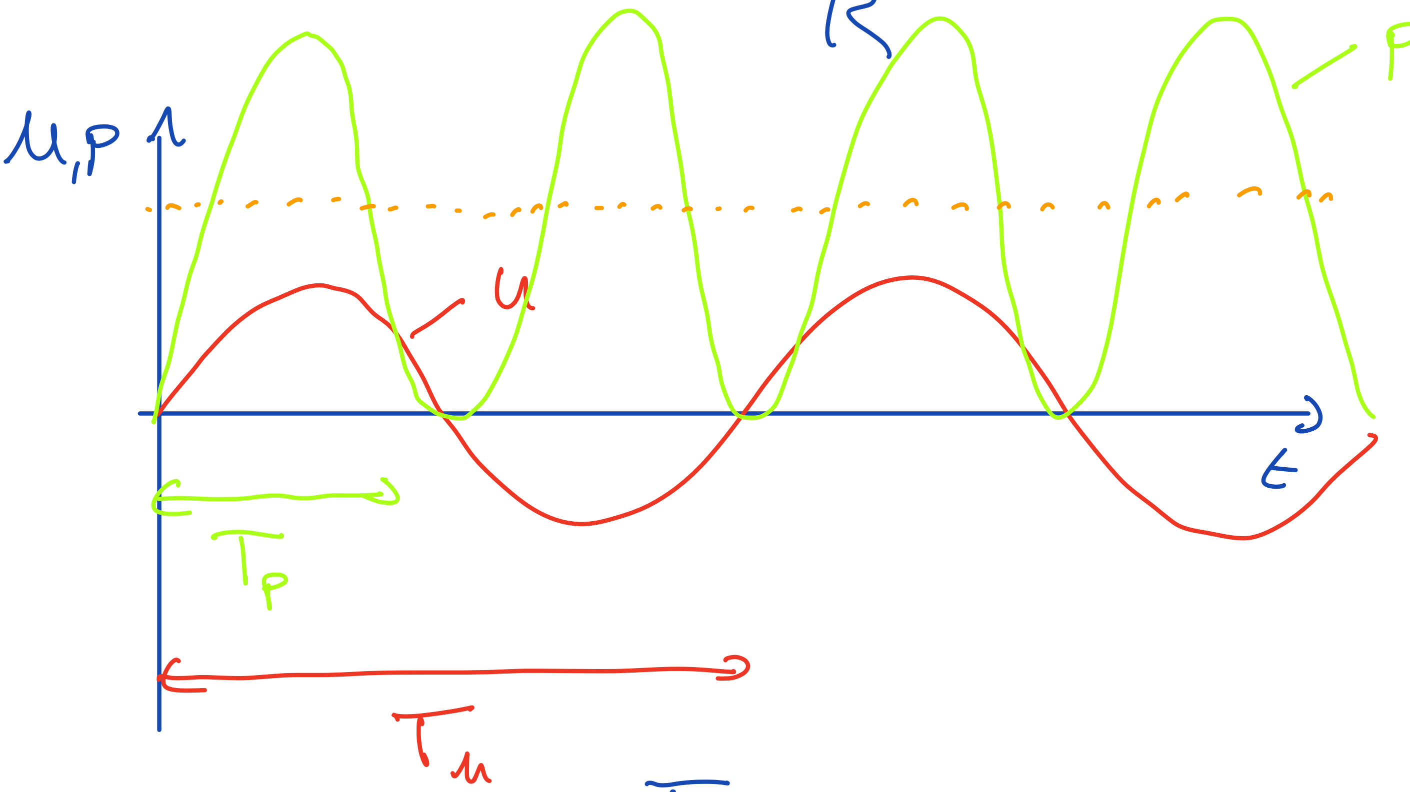
6.2.13 Puissance instantanée

$$p = u \cdot i$$

$$p = R \cdot i^2 = \frac{u^2}{R}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$= \frac{\hat{U}^2 \sin^2(\omega t + \alpha)}{R}$$



$$\bar{P}_R = \frac{1}{T} \int_0^T \frac{\hat{U}^2}{R} \sin^2(\omega t + \alpha)\, dt$$

$$\bar{P}_R = \frac{1}{T} \int_0^T \frac{\hat{U}^2}{R} \cos^2(\omega t + \alpha)\, dt$$

$$2\,\overline{P}_R = \frac{1}{T}\int_0^T \frac{\hat{u}^2}{R}\cdot \frac{1}{}\,dt$$

$$2\,\overline{P}_R = \frac{\hat{u}^2}{R} \rightarrow \overline{P}_R = \frac{\hat{u}^2}{2R}$$

En alternatif : $\overline{P}_R = \dfrac{\hat{u}^2}{2R}$

En continu : $P_R = \dfrac{u^2}{R}$

6.2.12 Valeur efficace :

(RMS Root Mean . Square)

Déf : $$\underset{\substack{\uparrow \\ \text{Majuscule}}}{U} = \sqrt{\frac{1}{T}\int_0^T \hat{u}^2 \underbrace{\sin^2(\omega t+\alpha)}_{\sin^2 x}\,dt}$$

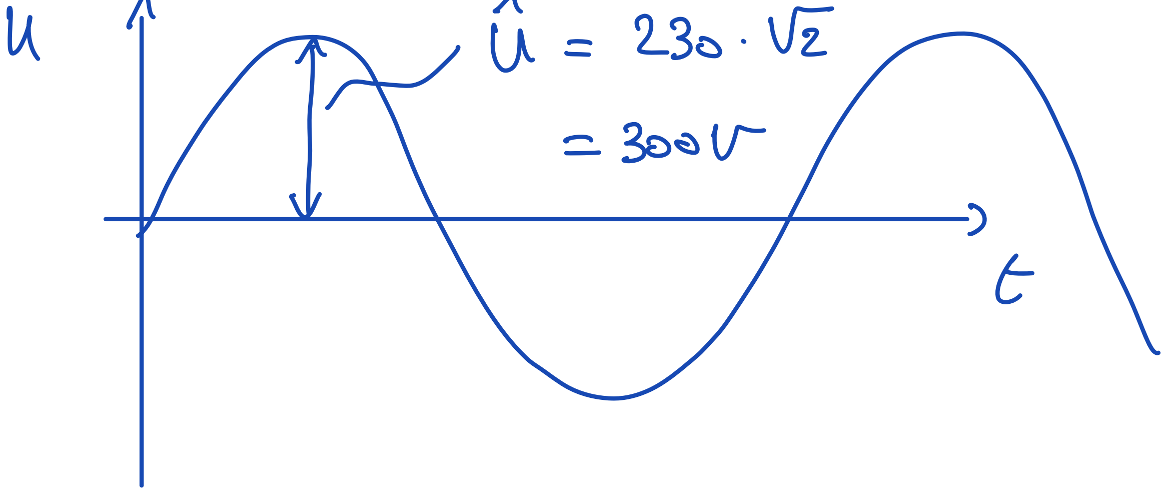
$$= \frac{1 - \cos 2\alpha}{2}$$

$$= \sqrt{\frac{\hat{u}^2}{T} \int_0^T \frac{1}{2}\,dt - \frac{1}{2}\frac{\hat{u}^2}{T} \underbrace{\int_0^T \cos(2\omega t - 2\alpha)\,dt}_{0}}$$

$$\boxed{u = \frac{\hat{u}}{\sqrt{2}}}$$

$$\underset{\text{crête}}{\hat{u}} = \underset{\text{efficace}}{u} \cdot \underset{\text{coef. avec sinus}}{\sqrt{2}}$$

$$\bar{P}_R = \frac{\hat{u}^2}{2R} = \frac{u^2}{R}$$



u, i valeurs instantanés

U, I valeurs efficaces

$\hat{U}, \hat{I}$ valeurs crêtes

$\bar{u}, \bar{i}$ valeurs moyennes

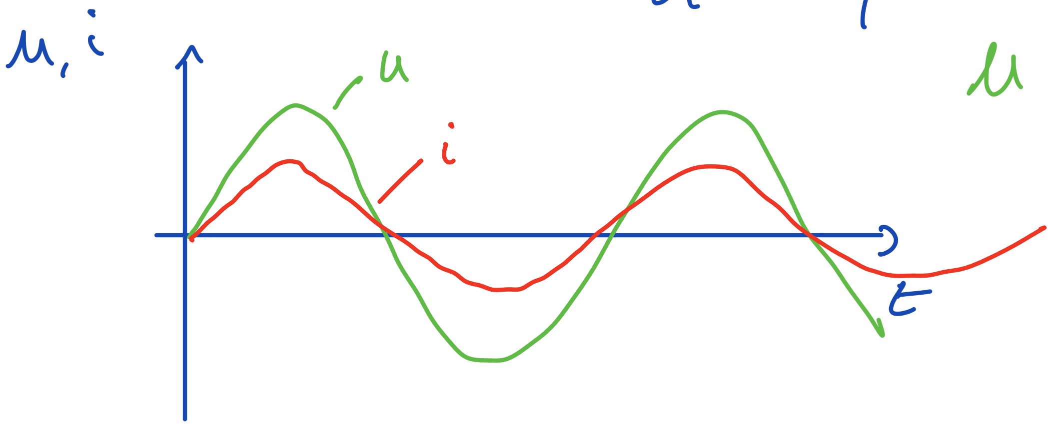
6.2.14 Cas de R

$$u = R \cdot i$$

$$\hat{U} \cos(\omega t + \alpha) = R \cdot \hat{I} \cos(\omega t + \beta)$$

Donc : $\hat{U} = R \cdot \hat{I}$

$\alpha = \beta$



u est en phase avec i

6.2.15 cas de L

$$u = L \frac{di}{dt}$$

$$\hat{U} \cos(\omega t + \alpha) = -\omega L \hat{I} \sin(\omega t + \beta)$$

$$= \omega L \hat{I} \cos\left(\omega t + \beta + \frac{\pi}{2}\right)$$

$$\hat{U} = \omega L \hat{I}$$

$$\alpha = \beta + \frac{\pi}{2}$$

Tension et le courant sont quadrature

retard du courant de $\frac{\pi}{2}$ sur la tension.

6.2.16 Cas de C

$$i = C \frac{du}{dt}$$

$$\hat{I} \cos(\omega t + \beta) = -\omega C \hat{U} \sin(\omega t + \alpha)$$

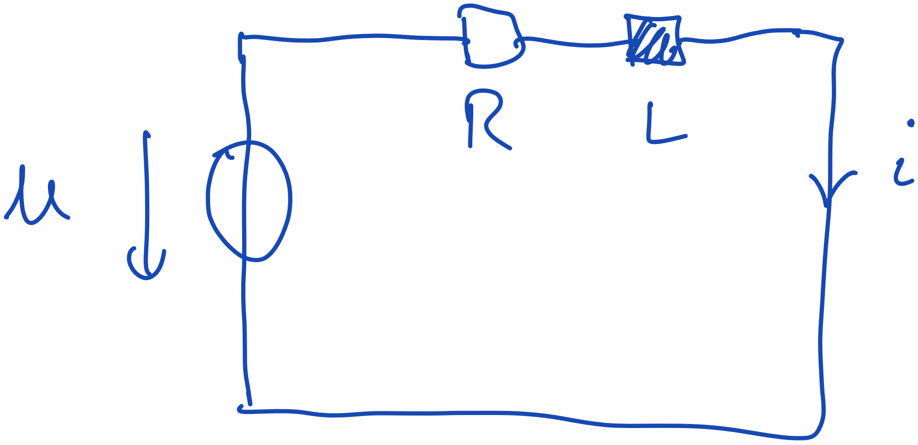
$$= \omega C \hat{U} \cos\left(\omega t + \alpha + \frac{\pi}{2}\right)$$

$$\hat{U} = \frac{\hat{I}}{\omega C}$$

$$\alpha = \beta - \frac{\pi}{2}$$

courant en avance de $\pi/2$ sur la Tension

6.3 Calcul complexe associé :



$u = u_R + u_L$

$u = \hat{U} \sin(\omega t + \alpha)$ connu

$i = \hat{I} \sin(\omega t + \beta)$ inconnu

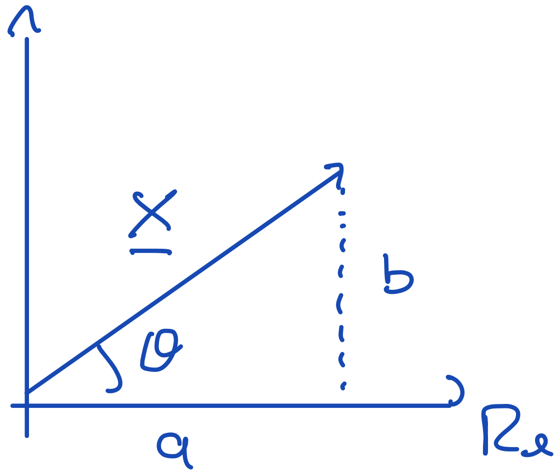
$\longrightarrow$ simplification : $\alpha = 0$

$\hat{U} \sin(\omega t) = R \cdot \hat{I} \sin(\omega t + \beta) +$

$\omega L \hat{I} \cos(\omega t + \beta)$

compliqué !!

Rappel : $j = \sqrt{-1}$



$$\underline{X} = a + bj$$

$$= \hat{X}\left(\cos\theta + j\sin\theta\right)$$

$$= \hat{X}\, e^{j\theta}$$

Concept :

$$u = \hat{U}\sin(\omega t) \xrightarrow{\text{Trans. compl.}} \underline{u} = \hat{U}\, e^{j(\omega t)}$$

Reel — imaginaire

$$u = \mathrm{Im}\{\underline{u}\}$$

$$\longleftarrow$$

$$i = \hat{I}\sin(\omega t + \beta) \longrightarrow \underline{i} = \hat{I}\, e^{j(\omega t + \beta)}$$

$$\longleftarrow$$

$$i = \mathrm{Im}\{\underline{i}\}$$

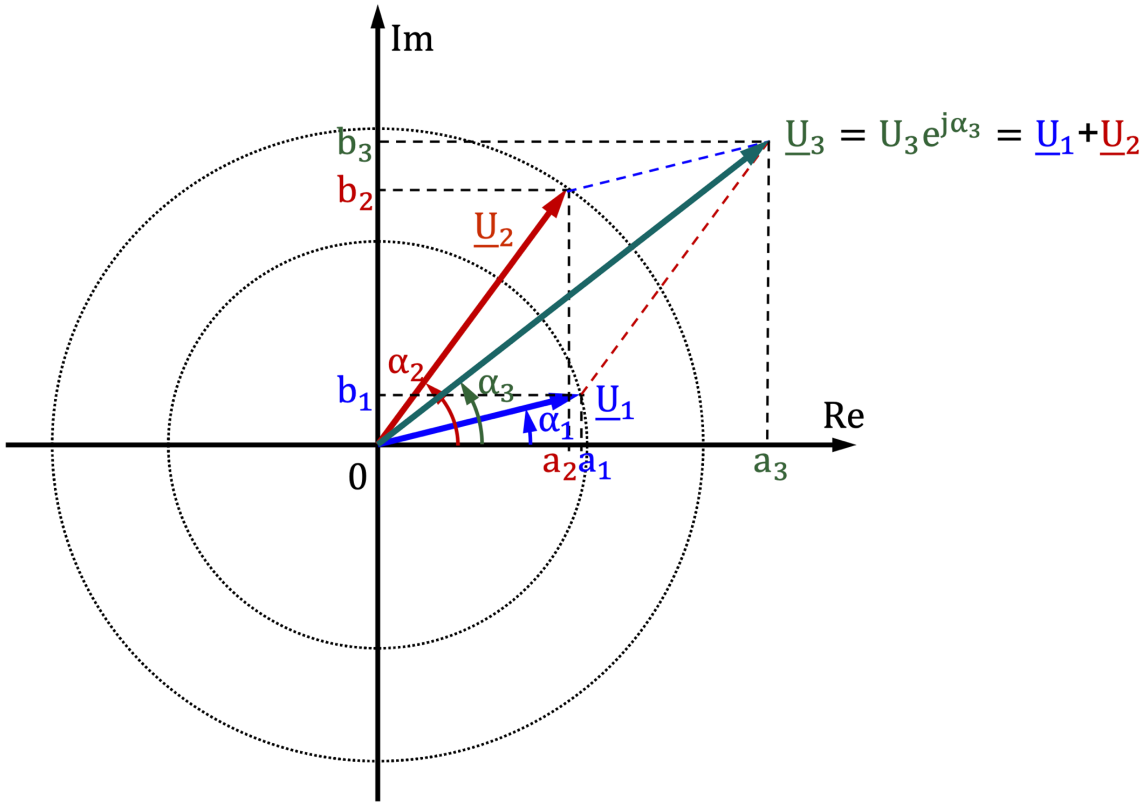
Année	2024-2025
Matière	Electrotechnique I
Questionnaire	Retour indicatif des enseignements (dès 2022-2023)
Nb Inscrit	309
Nb Répondu	212

Le déroulement du cours permet ma formation et un climat de classe approprié

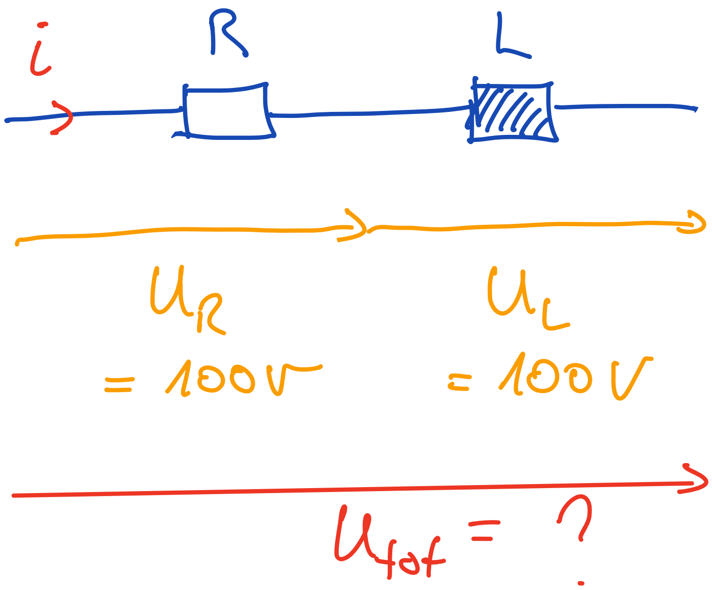
Sans avis	4%	9
Pas du tout d'accord	5%	10
Pas d'accord	13%	28
D'accord	51%	109
Tout à fait d'accord	26%	56

$$\begin{tikzcd}
u = \hat{U}\cos(\omega t + \alpha) \arrow[r, "\text{nb complexes}"] & \underline{u} = \hat{U}\, e^{j(\omega t+\alpha)} \arrow[d, "\text{on fait les calculs}"] \\
\text{résultats} & \text{résultats complexes} \arrow[l, "\mathrm{Re}\{\dots\}"']
\end{tikzcd}$$

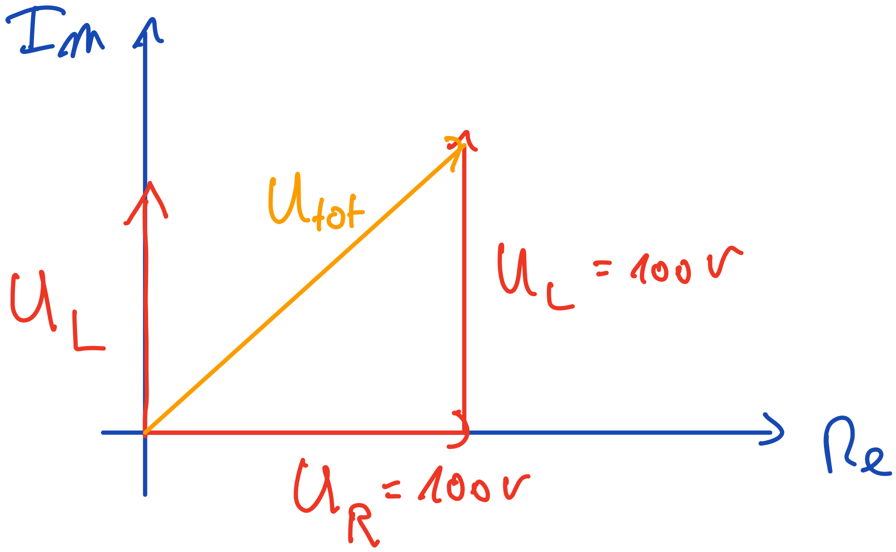
Diagramme des phaseurs



Exemple :



$$u = L \frac{di}{dt}$$

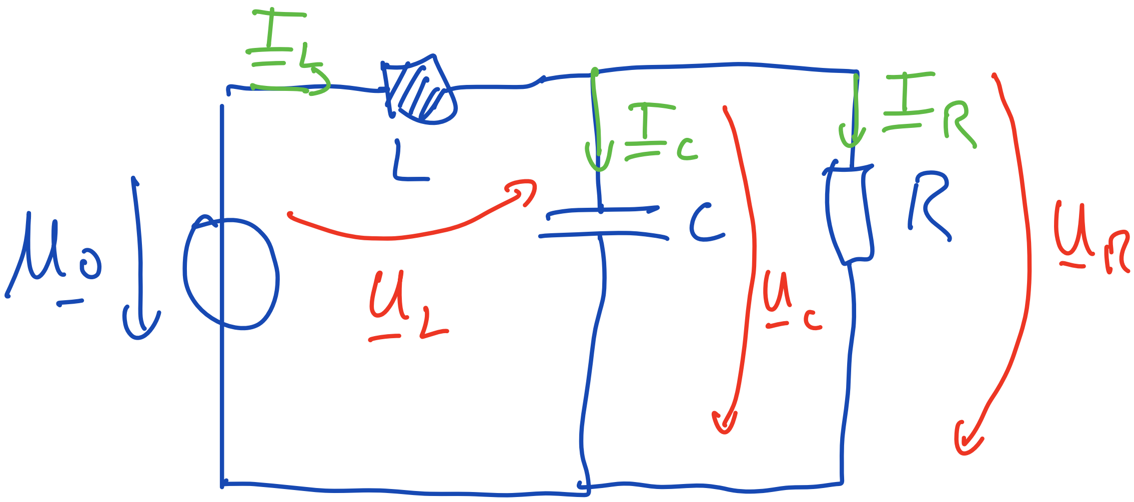


$$\underline{U}_L = j\omega L \underline{I}$$

$$|\underline{U}_{tot}| = \sqrt{U_R^2 + U_c^2}$$

$$= 141\ V$$

2ième Exemple :



$\underline{I}_c$ = connu $\qquad \underline{U}_0$?

$$\underline{U}_c = \underline{Z}_c \cdot \underline{I}_c = \frac{1}{j\omega C} \cdot \underline{I}_c = -\frac{j}{\omega C} \cdot \underline{I}_c$$

$$\frac{1}{j\omega C} \cdot \frac{j}{j} = \frac{-j}{\omega C}$$

$$\underline{U}_C = \underline{U}_R \qquad \underline{U}_R = R \cdot \underline{I}_R$$

$$\underline{I}_R = \frac{\underline{U}_C}{R} = -\frac{j}{\omega C R} \cdot \underline{I}_C$$

$$\underline{I}_L = \underline{I}_R + \underline{I}_C$$

$$\underline{I}_L = \underline{I}_C \left(1 - j \frac{1}{\omega C R} \right)$$

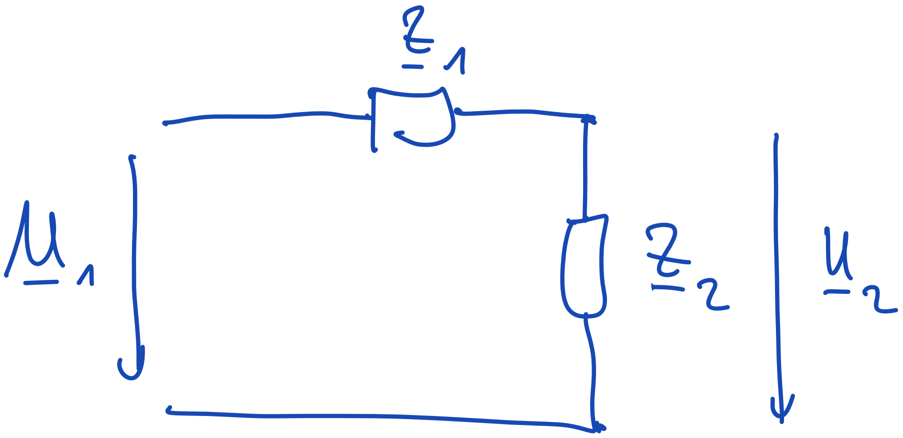
$$\underline{U}_L = \underline{Z}_L \cdot \underline{I}_L = j\omega L \, \underline{I}_L =$$

$$\underline{I}_C \left(\frac{L}{RC} + j\omega L \right)$$

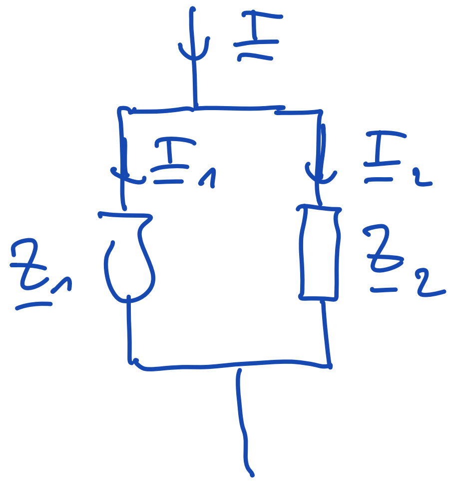
$$\underline{U}_0 = \underline{U}_L + \underline{U}_C = \underline{I}_C \left(\frac{L}{RC} + j\omega L \right) - j \frac{1}{\omega C} \underline{I}_C$$

$$= \underline{I}_C \left[\frac{L}{RC} + j \left(\omega L - \frac{1}{\omega C} \right) \right]$$

7.2.5 Division de tension et courant :



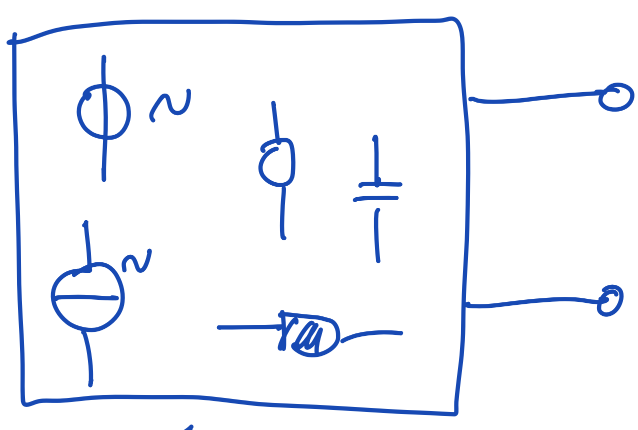
$$\underline{U}_2 = \frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2}\cdot\underline{U}$$

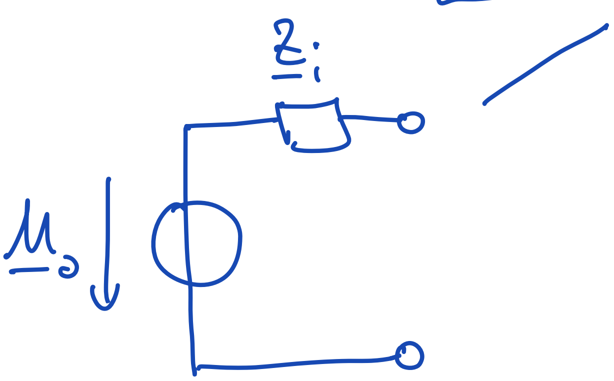


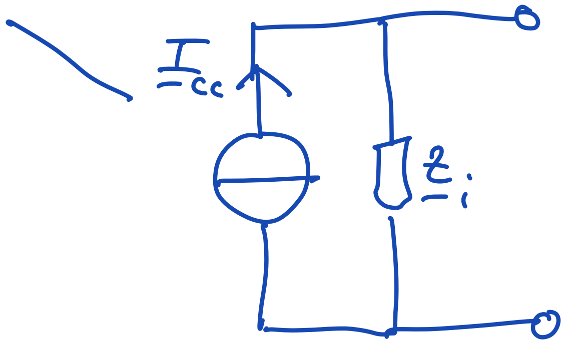
$$\underline{I}_2 = \frac{\underline{Z}_1}{\underline{Z}_1+\underline{Z}_2}\cdot\underline{I}$$

7.3.1 Théorèmes de Thévenin et Norton

Dipôle :







$\underline{U}_0$ = Tension à vide du circuit

$\underline{I}_{cc}$ = courant de court-circuit

$$\underline{Z}_i = \frac{\underline{U}_0}{\underline{I}_{cc}}$$

Sources de même fréquence

7.4 Principe de Superposition :

Système doit être linéaire

Cas No 1 Toutes les sources ont la même fréquence

On considère chaque source séparément : en annulant les autres :

Source No 1 : $\rightarrow \underline{I}_1$

Source No 2 : $\rightarrow \underline{I}_2$

⋮

$$\underline{I}_{tot} = \sum_{j=1}^{K} \underline{I}_j \qquad K = \text{nb de source}$$

valable pour $\underline{U}$ et $\underline{I}$

Cas No 2 : Les sources n'ont pas la même fréquence !

$$f_1 : \longrightarrow \underline{I}_{tot\,1}$$

$$f_2 : \longrightarrow \underline{I}_{tot\,2}$$

$$\vdots$$

$$\underline{I}_{tot\,1} \Rightarrow \underline{i}_{tot\,1} = \sqrt{2}\, I_{tot\,1}\, e^{j(\omega_1 t + \beta_1)}$$

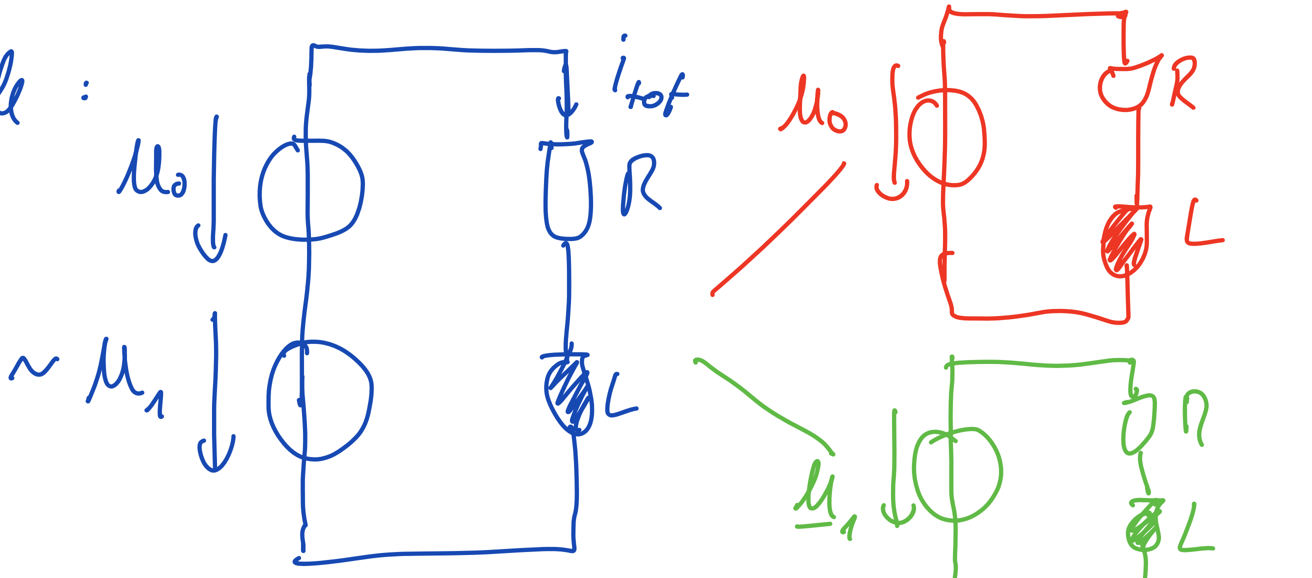
$$\underline{I}_{tot\,2} \Rightarrow \underline{i}_{tot\,2} = \sqrt{2}\, I_{tot\,2}\, e^{j(\omega_2 t + \beta_2)}$$

$$\vdots$$

$$\underline{i}_{tot} = \underline{i}_{tot\,1} + \underline{i}_{tot\,2}$$ (Monde temporel)

$$i_{tot} = \sqrt{2}\, I_{tot\,1} \sin(\omega_1 t + \beta_1) + \sqrt{2}\, I_{tot_2} \sin(\omega_2 t + \beta_2)$$

Exemple :

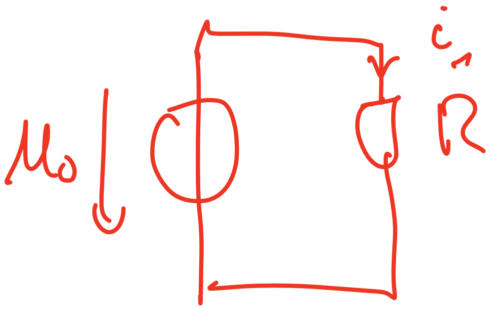


Cas de la source continue U_0 :

$$f = 0 \qquad \omega = 0$$

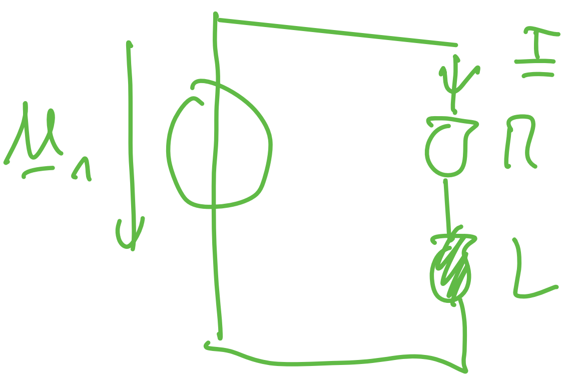
$$\underline{Z}_R = R$$

$$\underline{Z}_L = j\omega L = 0$$



$$i_1 = \frac{U_0}{R}$$

Cas de la source $\underline{U}_1$:



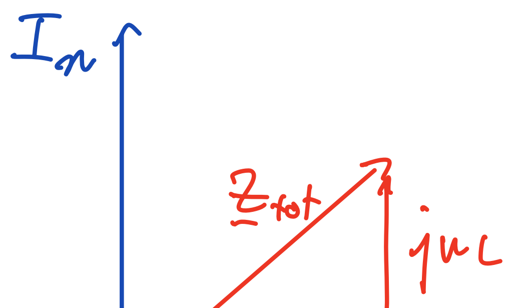
$$\underline{Z}_R = R$$

$$\underline{Z}_L = j\omega L$$

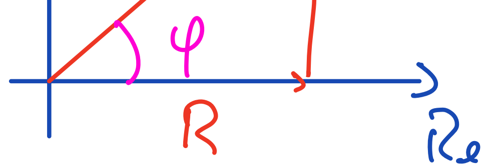
$$\underline{Z}_{tot} = R + j\omega L$$

$$\underline{U}_1 = \underline{Z}_{tot} \cdot \underline{I}_2$$

$$|\underline{Z}_{tot}| = \sqrt{R^2 + \omega^2 L^2}$$



$$\varphi = \text{Arctg}\,\frac{\omega L}{R}$$



$$\underline{U}_1 = U_1\, e^{j(0)} \qquad (\text{on pose } \alpha = 0)$$
$$= U_1$$

$$\underline{I}_2 = \frac{\underline{U}_1}{\underline{Z}_{tot}} = \frac{U_1}{\sqrt{R^2+\omega^2L^2}}\;\frac{e^{j0}}{e^{j\varphi}} = I_2\, e^{j-\varphi}$$

$$\Rightarrow$$

$$\underline{i}_2 = \sqrt{2}\, I_2\, e^{j(\omega t-\varphi)}$$

Monde réel : $i_2 = \sqrt{2}\, I_2 \sin(\omega t-\varphi)$

Résultat final : $i_{tot} = \frac{U_0}{R} + \sqrt{2}\, I_2 \sin(\omega t - \varphi)$

8. Puissances en alternatif sinus monophasé :

8.1 Puissance instantanée :

$$P(t) = p = u \cdot i$$

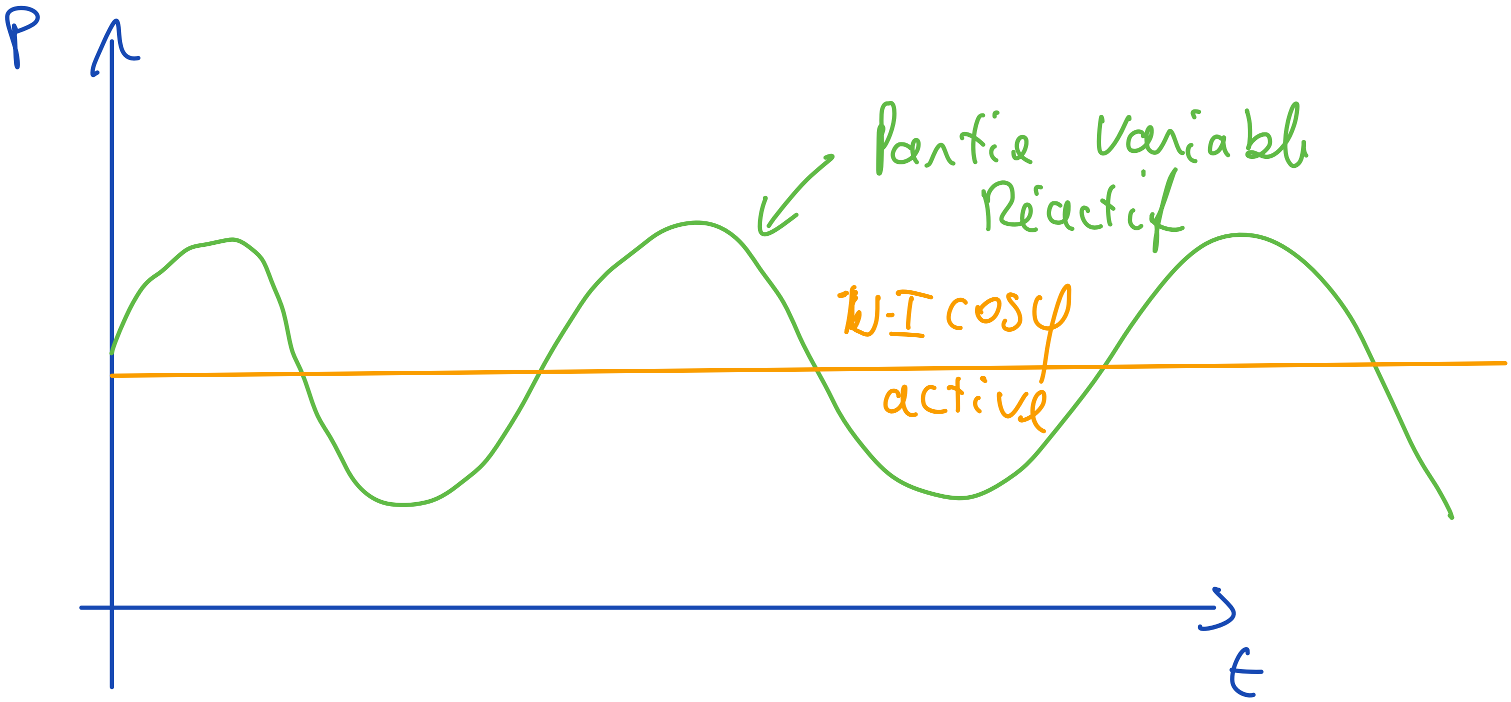
$$u = \hat{U} \cos(\omega t + \alpha)$$
$$i = \hat{I} \cos(\omega t + \beta)$$

$$p = \hat{U}\hat{I} \cos(\omega t + \alpha) \cos(\omega t + \beta)$$

$$\cos x \cdot \cos y = \frac{1}{2}\left(\cos(x-y) + \cos(x+y)\right)$$

$$p = \frac{\hat{U}\hat{I}}{2}\left[\cos\underbrace{(\alpha-\beta)}_{\varphi} + \cos(2\omega t + \alpha + \beta)\right]$$

* $$p = U \cdot I\left[\cos\varphi + \cos(2\omega t + \alpha + \beta)\right]$$



Impédance : $\underline{Z} = R + jX$

$R \downarrow$ Résistance

$X \downarrow$ réactance

$L \to \underline{Z}_L = j\omega L \to X_L$

On pose $\beta = \alpha - \varphi$

Identité : $\cos(2\omega t + 2\alpha - \varphi)$

$$= \cos\varphi \cos(2\omega t + 2\alpha) + \sin\varphi \sin(2\omega t + 2\alpha)$$

$$* \; p(t) = \underbrace{UI\cos\varphi\left[1 + \cos(2\omega t + 2\alpha)\right]}_{a}$$

$$+ \underbrace{UI\sin\varphi \sin(2\omega t + 2\alpha)}_{b} \qquad \text{Eq. } 8.3$$

a : oscille autour de $UI\cos\varphi$
toujours positif

b : oscille autour de 0, amplitude
moyenne est nulle

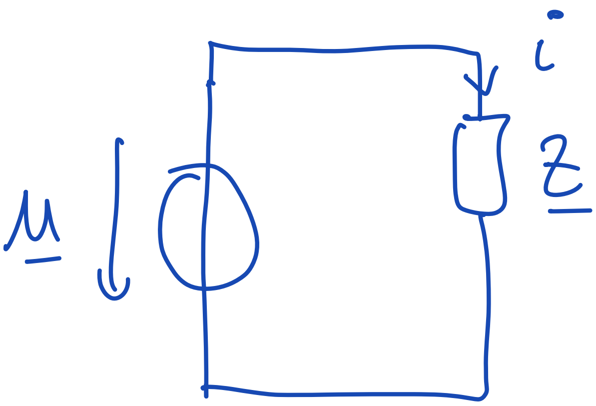
8.2 Puissance Active :

$$P = \overline{p}(t) = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= UI\cos\varphi \quad [W]$$

P = valeur moyenne de $p(t)$

= ce que l'on transforme et paye !



$$\varphi = \operatorname{Arctg} \frac{X}{R}$$

a) $\underline{Z} = R \rightarrow \varphi = 0$

$$P_R = UI\cos\varphi = UI$$

$$= \frac{\hat{U}\hat{I}}{2} = R \cdot I^2$$

b) Si $\underline{Z}_L = L$

$$\underline{Z}_L = j\omega L \qquad \varphi_L = \frac{\pi}{2}$$

$$P_L = UI\cos\varphi = 0$$

c) Si $\underline{Z}_c \Rightarrow C$

$$\underline{Z}_c = -\frac{j}{\omega C} \qquad \varphi_c = -\frac{\pi}{2}$$

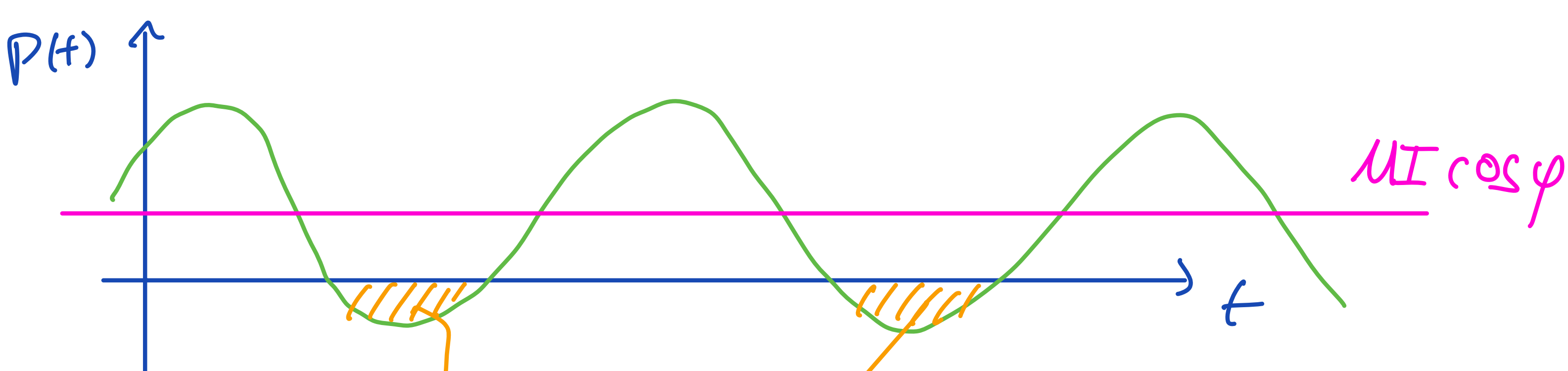
$$P_c = UI\cos\varphi = 0$$

8.3 Puissance Réactive :

Par définition, Amplitude de la composante Alternative de $p(t)$

Puissance fictive → caractérise l'échange de puissance non convertible

$$Q = UI\sin\varphi \quad [\mathrm{Var}]$$



φ rendu
à la source

Réactive positif : pour une inductance
" négatif : pour une capacité

Rappel : $P = UI \cos \varphi$

$Q = UI \sin \varphi$

8.4 Puissance Apparente :

$$S = U \cdot I \qquad [VA]$$

~ Volume de l'objet
~ prix de l'objet.

$$P = S \cos \varphi$$

$$Q = S \sin \varphi$$

$$S = UI$$

$$S = \sqrt{P^2 + Q^2}$$

8.5.3 Facteur de Puissance :

$$P = UI \underbrace{\cos\varphi}_{}$$

facteur de puissance

$$\cos\varphi = \frac{P}{UI}$$

8.5 Puissance apparente complexe

$$\underline{S} = P + jQ = Se^{j\varphi}$$

$$P = S\cos\varphi$$

$$Q = S\sin\varphi$$

$$S = UI = \sqrt{P^2 + Q^2}$$

$$P = UI \cos\varphi \qquad R \rightarrow P_R = R \cdot I^2$$

$$Q = UI \sin\varphi \qquad L, C \rightarrow Q_C = X \cdot I^2$$

réactance

Si L : $X = \omega L$

C : $X = -\frac{1}{\omega C}$

$$\underline{S} = \underbrace{RI^2}_{P} + \underbrace{jXI^2}_{Q}$$

8.5 - 4 - 6 :

R : $\underline{Z} = R \qquad \varphi = 0$

$$P_R = UI = RI^2 = \frac{U^2}{R}$$

$$Q_R = 0$$

$$S_R = UI = P_R$$

$$\cos\varphi = 1$$

L :

$$\underline{Z} = j\omega L \qquad \varphi = \frac{\pi}{2}$$

$$P_L = 0$$

$$Q_L = UI = XI^2 = \omega L I^2$$

$$S_L = UI$$

$$\cos\varphi = 0$$

C :

$$\underline{Z}_c = \frac{1}{j\omega C} = -\frac{j}{\omega C} \qquad \varphi = -\frac{\pi}{2}$$

$$P_c = 0$$

$$Q_c = -UI = -\frac{1}{\omega C} \cdot I^2$$

$$S_c = UI$$

$$\cos\varphi = 0$$

8.6 Résolution par les puissances :

Propriété :

$$P_{tot} = \sum_{k=1}^{n} P_k$$

n = nb de composants

$$Q_{tot} = \sum_{k=1}^{n} Q_k$$

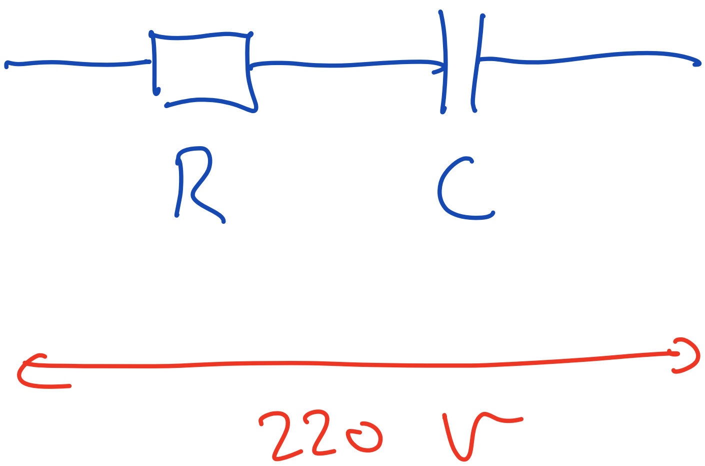
n = nb de composants

Mais : ⚠ $S_{tot} \neq \sum_{k=1}^{n} S_k$

Mais : $\underline{S}_{tot} = \sum_{k=1}^{n} \underline{S}_k$

(vectorielle)

Exemple :



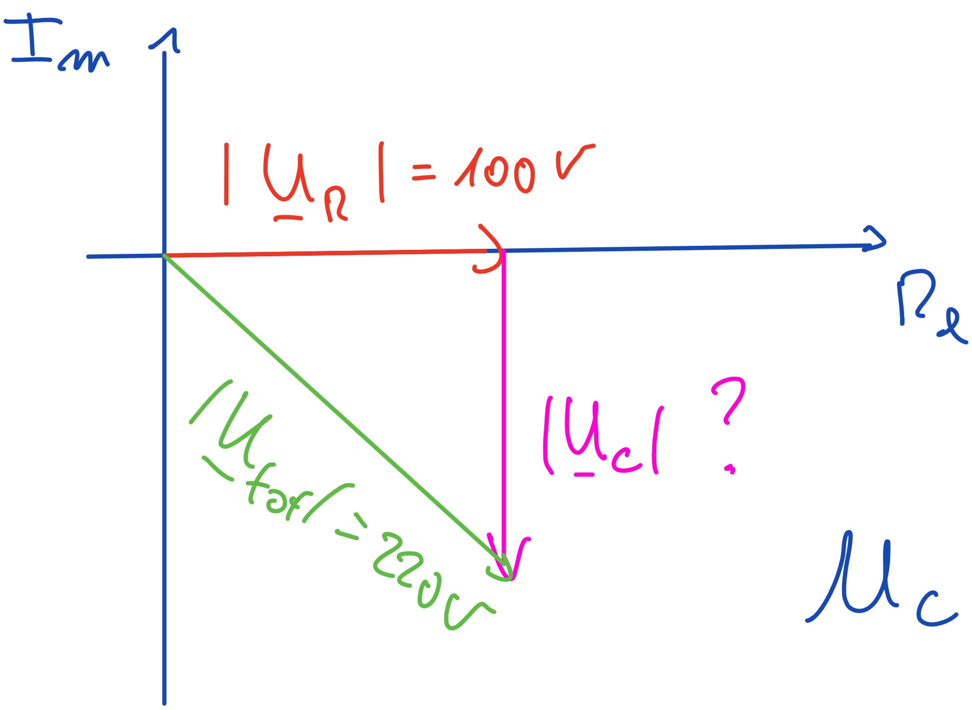
$f = 50\ Hz$

$U_R = 100\ V$

$P_{tot} = 500\ W$

U_c ?

$\underline{S}$?



$$U_c = \sqrt{U_{tot}^2 - U_R^2}$$

$$= 195.95\ V$$

$$\underline{S} = P + jQ$$

$$P = P_R = P_{tot} = 500\ W = RI^2$$

$$= \frac{U_R^2}{R}$$

$\rightarrow R = 20\ \Omega$

$\rightarrow I = 5A$

Résumé :

- Puissance instantanée : $p = u \cdot i$

- Puissance active : $P = U \cdot I \cos\varphi$

- Puissance réactive : $Q = UI \sin\varphi$

- Puissance Apparente : $S = UI$

(complexe) $\underline{S} = P + jQ$

Suite exemple :

$$P = S\cos\varphi$$

$$Q = S\sin\varphi$$

$$S = UI = \sqrt{P^2+Q^2}$$

$$P = UI\cos\varphi \qquad R \rightarrow P_R = RI^2$$

$$Q = UI\sin\varphi \qquad X \rightarrow Q = XI^2$$

$$\underline{S} = \underbrace{RI^2}_{P} + j\underbrace{XI^2}_{Q}$$

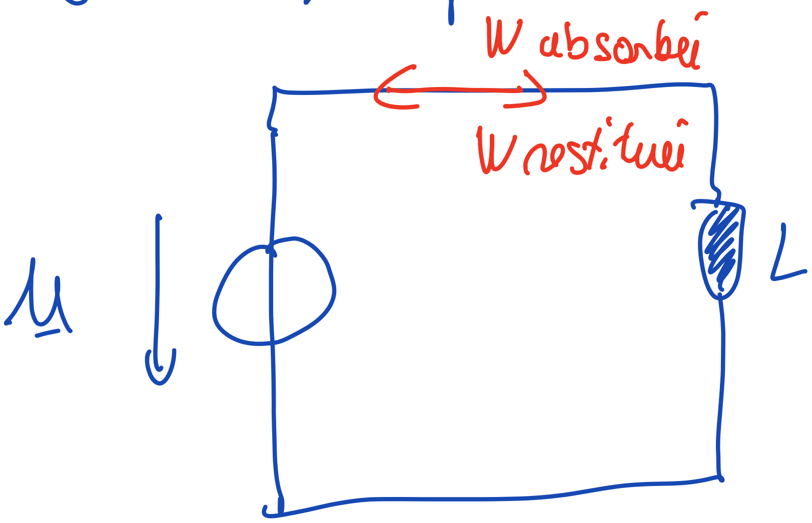
$$Q_c = -U_c \cdot I = -979{,}8 \text{ Var}$$

$$S = \sqrt{P^2+Q^2} = 1100 \text{ VA}$$

$$\varphi = \text{Artg}\ \frac{Q}{P} = -62.9^\circ$$

$$\cos\varphi = 0{,}45$$

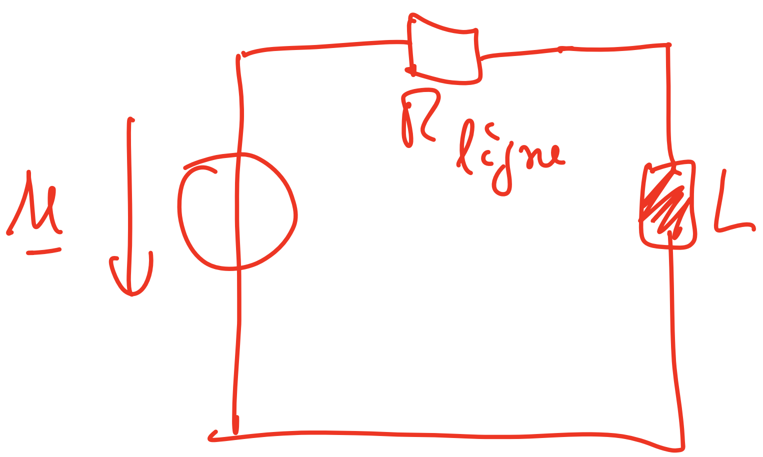
8.7 Adaptation de Puissance :



$P = 0$

$Q = U \cdot I$

Dans les faits :



$$P_{R_{ligne}} = R_\ell \cdot I^2$$

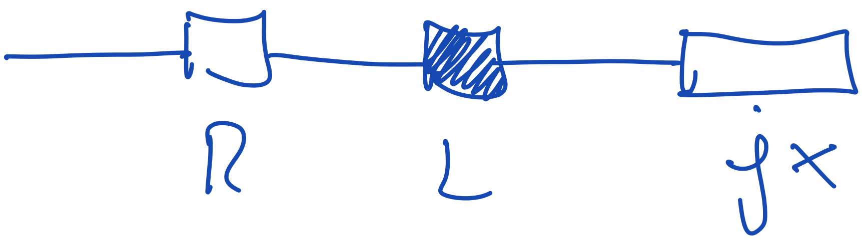
P_{active} : $\sim$ 25 ct / kWh

$Q_{réactif}$: ~ 6 ct/kvarh

→ Compensation du réactif en général → Capacité pour annuler/réduire la partie imaginaire de $\underline{Z}$

En général :

En série :



$$\underline{Z} = R + j\,(\omega L + X)$$

Si X est une capacité :

$$\underline{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

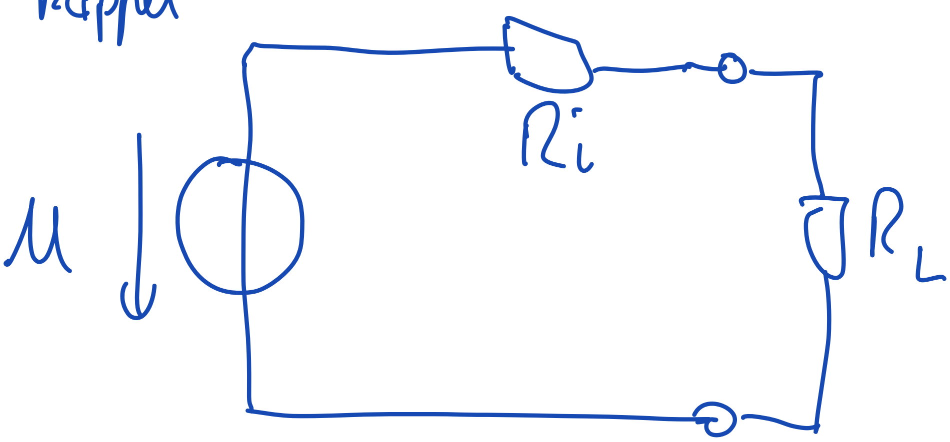
Condition de résonance :

Si $\omega L - \frac{1}{\omega C} = 0$

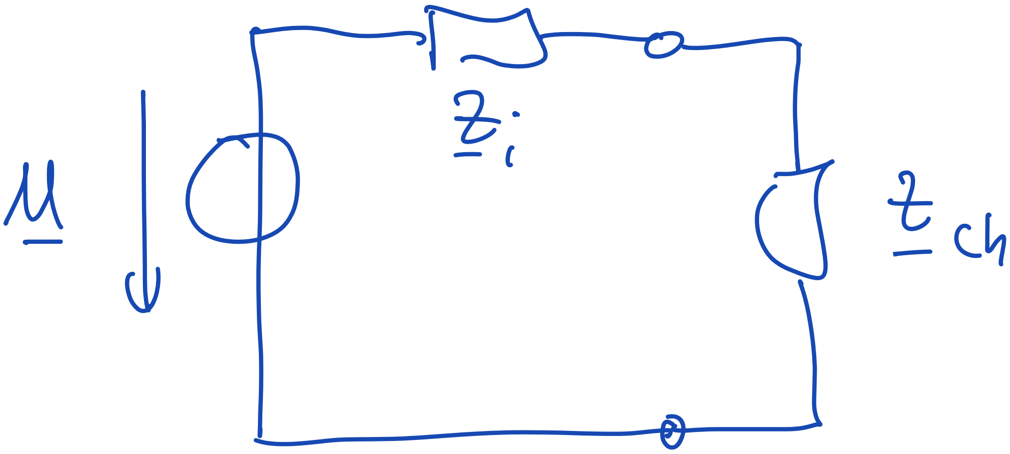
$$\omega = \sqrt{\frac{1}{LC}}$$

8.7.4 Adaptation d'une source de tension réelle :

Rappel



$P_{max} \Rightarrow R_i = R_L$



$$R_i = R_{ch}$$

$$X_i = -X_{ch}$$

$$\underline{Z}_i = R_i + jX_i$$

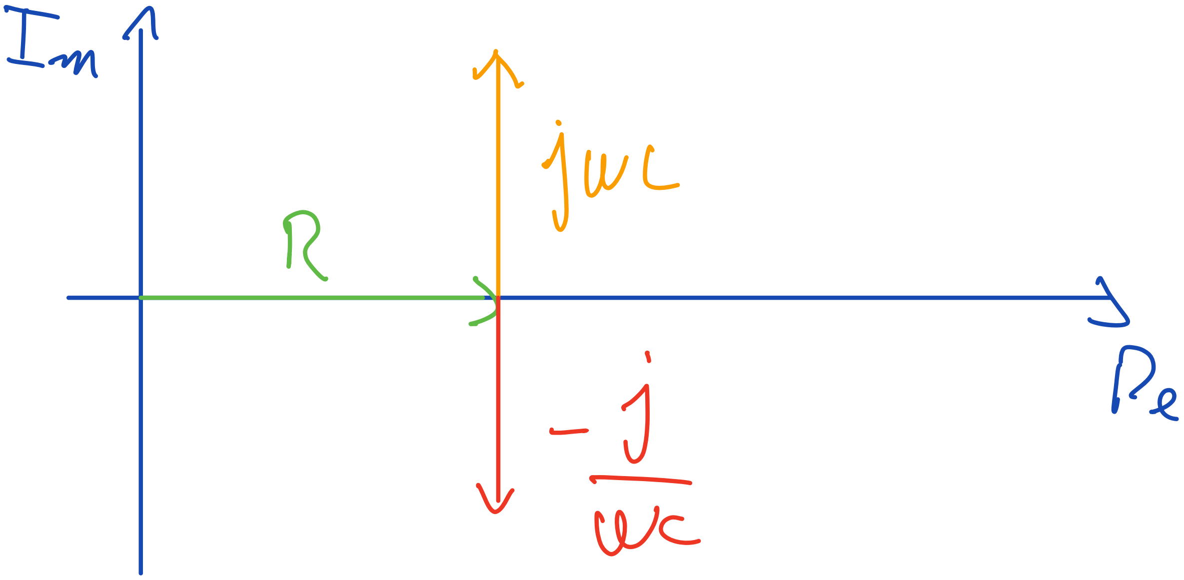
$$\underline{Z}_{ch} = R_{ch} + jX_{ch}$$

$$\rightarrow \underline{Z}_i = \underline{Z}_{ch}^*$$ (conjugué complexe)

9. Comportement fréquentiel :

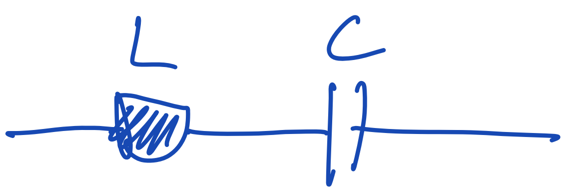
9.1 Lieux géométriques :

$$\underline{Z}_{tot} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$



9.2 Condition de résonance :

En série

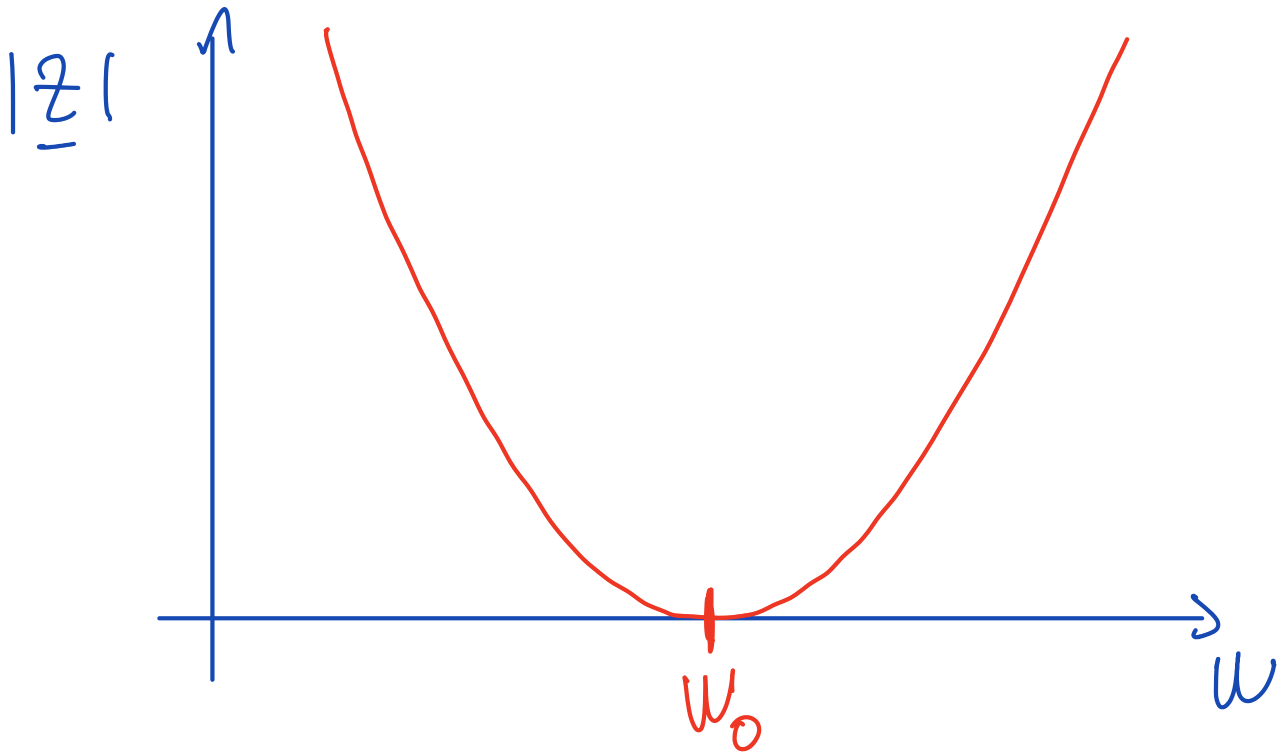


$$\underline{Z}_{eq} = j\left(\omega L - \frac{1}{\omega C}\right)$$

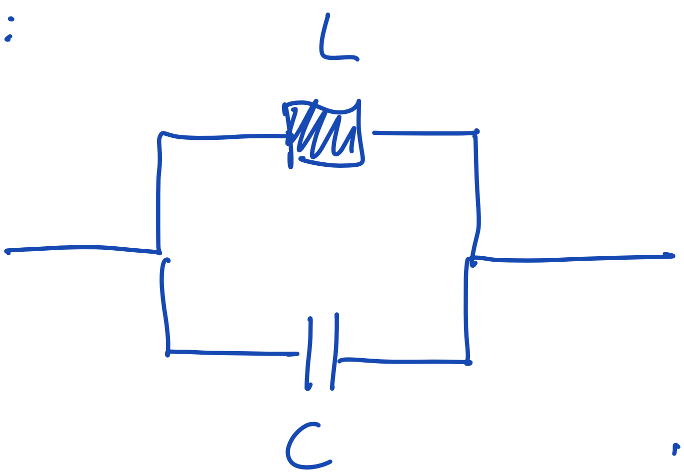
$$= j\,\frac{\omega^2 LC - 1}{\omega C}$$

Si $\underline{Z}_{eq} = 0$ $\rightarrow$ condition de résonance

$$\omega^2 LC - 1 = 0 \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$



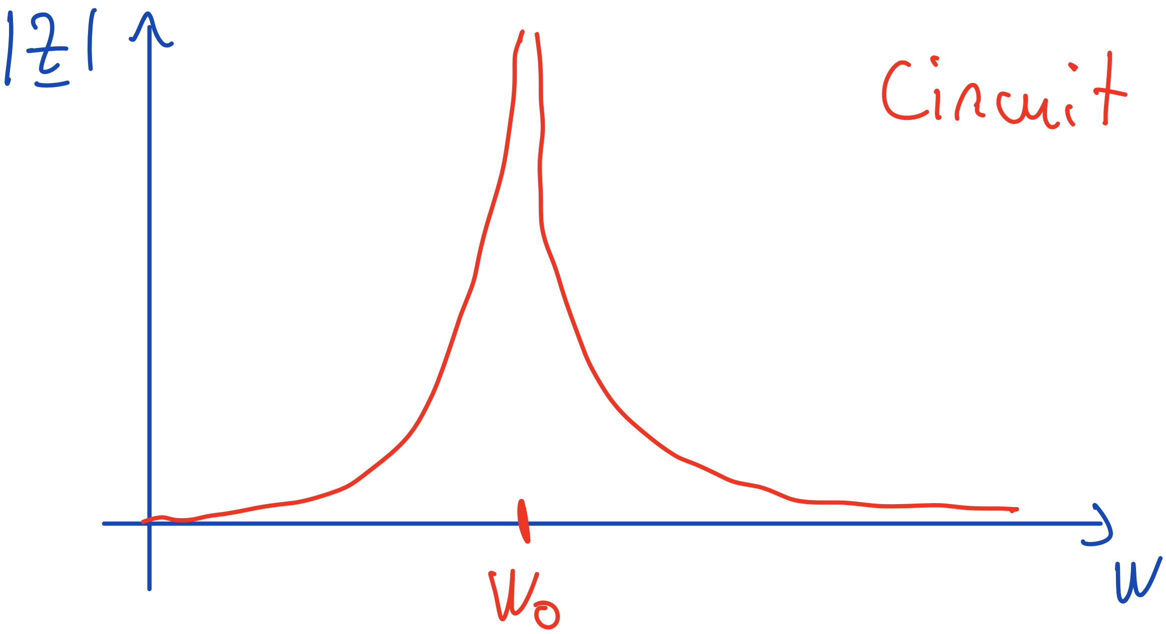
En // :



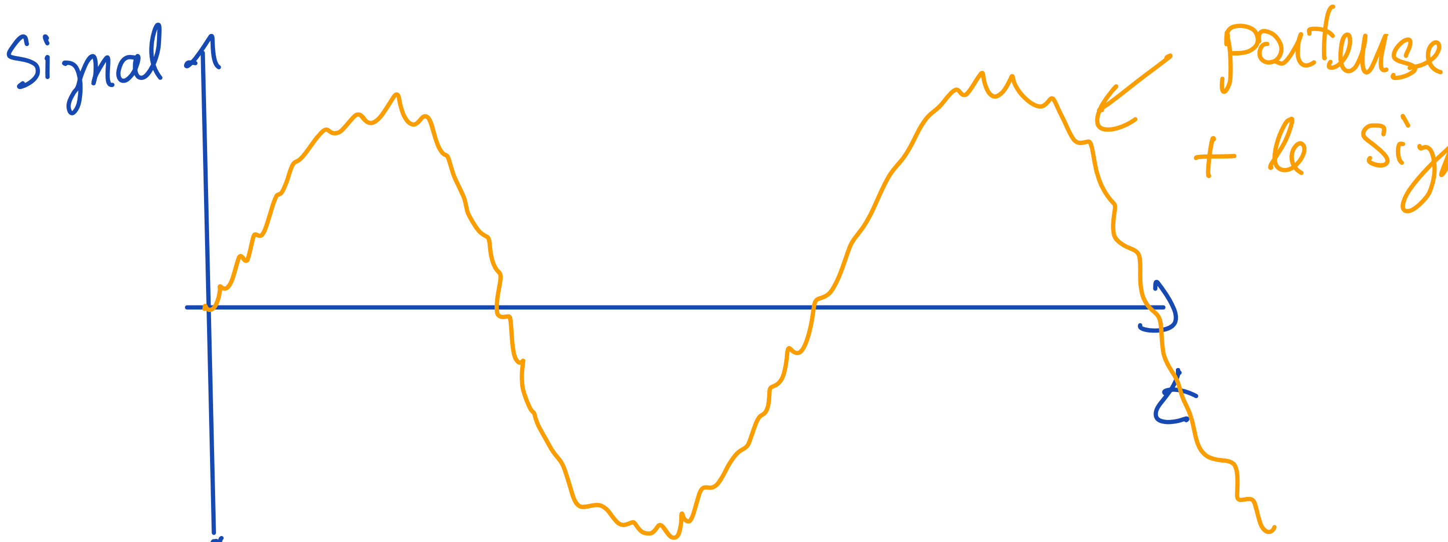
$$\underline{Z}_{eq} = \frac{1}{\frac{1}{j\omega L} + j\omega C} = \frac{j\omega L}{1 - \omega^2 LC}$$

Si $\omega_0 = \sqrt{\frac{1}{LC}}$ $\quad (1 - \omega^2 LC = 0)$

$\hookrightarrow \underline{Z}_{eq} \to \infty$



Circuit bouchon !



Corrigé Exo Auto-évaluation :